

## TOPIC:Function

- 1) Define even and odd function
- 2) If  $f(x) = x^3 - 5x^2 - 4x + 20$  then show that  $f(0) = -2f(3)$
- 3) If  $f(x) = x^4 - 2x + 7$  find  $f(0) + f(2)$
- 4) If  $f(x) = x^2 + 3$  then find the value of  $x$  for which  $f(x) = f(2x+1)$
- 5) Show that  $3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x$  is even function.
- 6) If  $f(x) = \sin x$   $g(x) = \cos x$  show that  $f(x+y) = f(x)g(y) + g(x)f(y)$
- 7) If  $f(x) = \cos x$  show that  $f(3x) = 4f^3(x) - 3f(x)$
- 8) If  $f(x) = \tan x$  show that  $f(2x) = \frac{2f(x)}{1-f(x)^2}$
- 9) If  $f(x) = 50\sin(100\pi t + 0.4)$  show that  $f(\frac{1}{50} + t) = f(x)$
- 10) If  $f(x) = \log(\frac{x}{x-1})$  then prove that  $f(a+1) + f(a) = \log(\frac{a+1}{a-1})$
- 11) If  $f(x) = \frac{x+5}{3x-4}$  and  $t = \frac{5+4x}{3x-1}$  then show that  $f(t) = x$
- 12) If  $y = f(x) = \frac{x+1}{x-1}$  then show that  $x = f(y)$
- 13) If  $f(x) = \log(\frac{1+x}{1-x})$  then prove that  $f(\frac{2x}{1+x^2}) = 2f(x)$
- 14) If  $f(x) = \frac{2x+5}{3x-4}$  and  $g(x) = \frac{5+4x}{3x-2}$  then show that  $f \circ g(x) = x$
- 15) If  $y = f(x) = \frac{2x-3}{3x-2}$  then show that  $x = f(y)$
- 16) If  $f(x) = \frac{x+3}{4x-5}$  and  $t = \frac{3+5x}{4x-1}$  then show that  $f(t) = x$
- 17) if  $f(x) = \log x$ ,  $\phi(x) = x^3$  show that  $f[\phi(x)] = 3f(x)$ .
- 18) if  $f(x) = 3x^4 + x^2 + 5 - 3\cos x + 3\sin^2 x$ , show that  $f(x) + f(-x) = 2f(x)$ .
- 19) if  $f(x) = x^2 + 4$ , solve  $f(x+1) - f(x-1) - 12 = 0$ .
- 20) if  $f(x) = x - \frac{1}{x}$  then prove that  $[f(x)]^3 = f(x^3) + 3f(\frac{1}{x})$ .
- 21) if  $f(x) = \log(\frac{1+x}{1-x})$  then show that  $f(a) + f(b) = f(\frac{a+b}{1+ab})$ .
- 22) if  $f(x) = 16^x + \log_4 x$  then find  $f(\frac{1}{2})$  and  $f(\frac{1}{4})$ .
- 23) if  $f(x) = \frac{x^2+9}{\sqrt{x-3}}$   $f(4) + f(5)$ .
- 24) if  $f(x) = \log(x + \sqrt{1+x^2})$ , show that the function is odd.

## TOPIC: DERIVATIVE'S

- 25) a) Using first principle of derivative find derivative of following functions:
  - 26) 1.  $e^x$       2.  $3x^2+4$       3.  $\sqrt{x}$       4.  $\log x$       5.  $\sin x$       6.  $\cos x$       7.  $\tan x$       8. K.
- 27) b) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = u + v$ , then prove that:  $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ .
- 28) c) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = u - v$ , then prove that:  $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ .
- 29) d) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = u \cdot v$ , then prove that :  $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ .
- 30) e) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = \frac{u}{v}$ , then prove that :  $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ .
- 31) f) Find  $\frac{dy}{dx}$ , if i)  $y = x^{10} + 10^x + e^x$  ii)  $y = (x+1)(x+2)$ . iii)  $y = \sec x \cdot \tan x$  iv)  $y = e^x \cdot \tan x$

32) v)  $y = \frac{x+1}{x-1}$  vi)  $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$  vii)  $y = \frac{\sin x}{1-\cos x}$  . viii)  $y = \frac{\sin x + \cos x}{\cos x - \sin x}$  ix)  $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$  .

33) f) Differentiate w. r. t. x

34) i)  $\sin(x^2 + 5)$  . ii)  $\sec(4x - 3)$  iii)  $e^{3x} \cos 2x$  iv)  $\log(\sin x)$  v)  $\tan(x \cdot e^x)$  vi)  $(x^2 + x + 1)^4$

35) g) Find  $\frac{dy}{dx}$  if i)  $y = \log(3x^2 + 2x + 1)$  ii)  $y = e^{\tan x}$  iii)  $y = \sin^2[\log(2x + 3)]$  iv)  $y = \log(\log x)$ .

36) v)  $y = \cos(x^2 e^x)$  vi)  $y = \cos^{-1}(7x)$  vii)  $y = e^{3x} + \tan^4 x$  viii)  $y = e^{\sin x} \cos 4x$ .

37) h) Find  $\frac{dy}{dx}$  if  $x^3 + y^3 + 4x^3y = 0$

38) i) Find  $\frac{dy}{dx}$  if  $x^3 + y^3 = 4xy$

39) j) Find  $\frac{dy}{dx}$  if  $x^2 + 3y + y^2 = 5$  at (1,1) .

40) k) Find  $\frac{dy}{dx}$  if  $x^2 + y^2 = 4xy$  .

41) l) Find  $\frac{dy}{dx}$  , if  $\sin y = \log(x + y)$

42) m) Find  $\frac{dy}{dx}$  if  $x^2 + y^2 + xy - y = 0$  at (1,2) .

43) n) if  $x^3 + y^3 = 3axy$  , find  $\frac{dy}{dx}$  at the point  $(\frac{3a}{2}, \frac{3a}{2})$  .

44) o) if  $x^y = e^{x-y}$  , then prove that  $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

45) p) if  $e^y = e^x$  , then find  $\frac{dy}{dx}$

46) q) Find  $\frac{dy}{dx}$  if  $x^2 + y^2 = 25$  .

47) r) Find  $\frac{dy}{dx}$  if  $x^2 + 3xy - y^2 = 11$  at (1,2) .

48) a) if  $y = e^{2x} \cdot \sec x \cdot \tan 2x$  , Find  $\frac{dy}{dx}$  .

49) b) i) if  $y = \frac{(x+1)^4}{(x-1)^3(x+4)^7}$  , Find  $\frac{dy}{dx}$  . ii) if  $y = \frac{(x-2)^3(x-3)^5}{(x+1)^4}$  , Find  $\frac{dy}{dx}$  .

50) c) Find  $\frac{dy}{dx}$  , if  $y = x^x$  .

51) d) Differentiate  $(\sin x)^{\tan x}$  w. r. to. x .

52) e) Differentiate  $(\tan x)^{\cot x}$  w. r. to. x .

53) f) Differentiate w. r. to. x . i)  $(\log x)^{\log x}$  ii)  $(\sin x)^{\cos x}$  .

54) a) Find  $\frac{dy}{dx}$  , if  $x = 3at^2$  and  $y = 2at^3$  .

55) b) Find  $\frac{dy}{dx}$  , if  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$  .

56) c) Find  $\frac{dy}{dx}$  , if  $x = \frac{2t}{1+t^2}$  and  $y = \frac{1-t^2}{1+t^2}$  .

57) d) Find  $\frac{dy}{dx}$  , if  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$  .

58) e) Find  $\frac{dy}{dx}$  , if  $x = at^2$  and  $y = 2at$  .

59) f) Find  $\frac{dy}{dx}$  , if  $x = a(2\theta - \sin 2\theta)$  and  $y = a(1 - \cos 2\theta)$  .

60)

61) a) Differentiate  $\log(x \cdot \sin x)$  w. r. to  $\frac{1}{x}$  .

62) b) Differentiate  $\tan^{-1}(4x)$  w. r. to  $e^{6x}$  .

63) c) Differentiate  $(\log x)^x$  w. r. to  $5^{4x}$  .

64) d) Differentiate  $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$  w. r. to  $\sin^{-1}(2x \cdot \sqrt{1-x^2})$  .

65) e) Differentiate  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$  w. r. to  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$  .

- 66) f) Differentiate  $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$  w.r. to  $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ .
- 67) a) if  $xy = a$ , show that  $x \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$ .
- 68) b) if  $y = \sin 5x - 3 \cos 5x$ , show that  $\frac{d^2y}{dx^2} + 25y = 0$ .
- 69) c) if  $y = \sin^{-1} x$ , show that  $(1 - x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$ .
- 70) d) if  $y = 2 \cos(\log x) + 3 \sin(\log x)$ , show that  $x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + y = 0$ .
- 71) e) if  $y = e^{m \cdot \sin^{-1} x}$ , show that  $(1 - x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$ .
- 72) f) if  $y = ae^{2x} + be^{-2x}$ , show that  $\frac{d^2y}{dx^2} - 4y = 0$ .
- 73) g) if  $y = A \cos x + B \sin x$ , show that  $\frac{d^2y}{dx^2} + y = 0$ .
- 74) h) if  $y = \cos(\log x)$ , show that  $x^2 \cdot \frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} - y = 0$ .
- 75) i) if  $y = 3 \sin 4x - 5 \cos 4x$ , show that  $\frac{d^2y}{dx^2} + 16y = 0$ .
- 76) j) if  $y = a \cos(\log x) + b \sin(\log x)$ , show that  $x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + y = 0$ .

### TOPIC: Application of Derivative

#### -Equation of tangent and normal→

- At which point on curve  $y = 3x - x^2$  the slope is -5.
- Find the points on curve  $y = 7x - x^2$  where the inclination of tangent is  $45^\circ$ .
- Find the inclination of the tangent to the curve  $y = e^{2x}$  at  $(1, -3)$ .
- Find the point on curve  $y = \log(x - 3)$  at which slope is 5.
- Find the point on the curve  $y = x^3 - 3x + 3$ , the tangent at which is parallel to x-axis.
- The slope of the curve  $2y^3 = ax^2 + b$  at  $(1, -1)$  is same as the slope of  $x + y = 0$ , Find a and b.
- Find the equation of tangent and normal to the curve  $y = x(2 - x)$  at point  $(2, 0)$ .
- Find the equation of tangent and normal to the curve  $y = x^3 - 2x^2 + 4$  at  $x = 2$ .
- Find the equation of tangent and normal to the curve  $x^2 + 3xy + y^2 = 5$  at  $(1, 1)$ .
- Find the equation of tangent and normal to the curve  $4x^2 + 9y^2 = 40$  at  $(1, 2)$ .
- Find the equation of tangent and normal to the curve  $13x^3 + 2x^2y + y^3 = 1$  at  $(1, -2)$ .
- Find the equation of tangent to the curve  $y = x^2 - 2x - 3$  where it cuts the X-axis.
- Show that the equation of tangent to the curve  $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2$  at point  $(a, b)$  is  $\frac{x}{a} + \frac{y}{b} = 2$ .
- Find the equation of tangent to the curve  $y = 9x^2 - 12x + 7$  which is parallel X-axis.

#### -Maxima and Minima-

1. Find the maximum and minimum values of  $y = x^3 - 9x^2 + 24x$ .
2. Find the maximum and minimum values of  $y = 2x^3 - 9x^2 + 12x + 5$ .
3. Find maxima and minima of  $y = 2x^3 - 3x^2 - 36x + 10$ .
4. Find the values of  $x$  for which function is maximum and minimum if  $y = x^3 - \frac{15}{2}x^2 + 18x$ .
5. Find the maximum and minimum values of  $y = x^3 - 18x^2 + 96x$ .
6. Divide 80 into two parts such that their product is maximum.
7. A metal wire 36 cm long is bent to form a rectangle. Find its dimensions, when its area is maximum.
8. A manufacturer can sell  $x$ -items at the rate of Rs.  $(330 - x)$  each. The cost of producing  $x$ -items in Rs.  $x^2 + 10x + 12$ . How many items must be sold so that his profit is maximum.
9. A manufacturer can sell  $x$ -items at the rate of Rs.  $(23 - 0.001x)$  each. The cost of producing  $x$ -items in Rs.  $(5x + 2000)$ . How many items must be sold so that his profit is maximum.

#### -Radius of Curvature-

1. Find the radius of curvature to the curve  $y = x^3$  at  $(2, 8)$ .
2. Find the radius of curvature to the curve  $y = x^3 + 3x^2 + 2$  at  $(1, 2)$ .
3. Find the radius of curvature of the curve  $y = \log(\sin x)$  at  $x = \frac{\pi}{2}$ .
4. Find the radius of curvature of the curve  $x^2 + y^2 = 25$  at point  $(4, 3)$ .
5. Find the radius of curvature of the curve  $\sqrt{x} + \sqrt{y} = 1$  at point  $(\frac{1}{4}, \frac{1}{4})$ .
6. A beam is bent in the form of a curve  $y = 2 \sin x - \sin 2x$ . Find the radius of curvature of the beam at point  $= \frac{\pi}{2}$ .
7. Find the radius of curvature of  $y = a \log\left(\sec \frac{x}{a}\right)$ .
8. Find the radius of curvature of  $x = \cos^3 \theta$ ,  $y = \sin^3 \theta$  at  $= \frac{\pi}{4}$ .

#### TOPIC: Integration

- 1.Evaluate:** 1)  $\int (e^x + x^e + e^e) dx$  2)  $\int (x^a + a^x + a^a) dx$  3)  $\int e^{2x} dx$  4)  $\int (5^x + \sec^2 x) dx$   
 5)  $\int \left(\frac{1}{\sqrt{1-x^2}} - \cos x\right) dx$  6)  $\int \left(\frac{1}{1+x^2} + 5^x\right) dx$  7)  $\int \left(\frac{1}{1+x^2} - \frac{\cos x}{\sin^2 x}\right) dx$  8)  $\int \frac{1}{\sqrt{x}} dx$   
 9)  $\int (\tan x + 4 \sin x) dx$  10)  $\int \sin x \cdot \sec^2 x dx$  11)  $\int \frac{\sec x \cdot \operatorname{cosec} x}{\tan x} dx$  12)  $\int \frac{1}{1+\sin x} dx$   
 13)  $\int x \cdot (x-1)^2 dx$  14)  $\int \frac{x^2-4x+5}{x} dx$  15)  $\int \sqrt{x}(1+x) dx$  16)  $\int \sqrt[3]{x}(x^2+x-1) dx$

$$17) \int \frac{dx}{\sqrt{x-1}} \quad 18) \int (x+1)^2 dx \quad 19) \int \left( \frac{1}{1+x^2} + e^{4x} \right) dx.$$

$$2. \text{Evaluate : } 1) \int \frac{1}{25+x} dx \quad 2) \int \frac{1}{2x+1} dx \quad 3) \int \frac{1}{2x+11} dx \quad 4) \int \frac{dx}{4-x^2} \quad 5) \int \frac{1}{3x^2+4} dx \quad 6) \int \frac{1}{2x^2+3} dx$$

$$7) \int \frac{1}{4x^2+25} dx \quad 8) \int \frac{1}{4x^2-9} dx \quad 9) \int \frac{1}{4-9x^2} dx \quad 10) \int \frac{1}{25-9x^2} dx \quad 11) \int \frac{dx}{\sqrt{4x^2+9}} \quad 12) \int \frac{dx}{\sqrt{4x^2+25}}$$

$$13) \int \frac{dx}{\sqrt{4x^2-9}}.$$

$$3. \text{Evaluate : } 1) \int \frac{\sin^2 x}{1+\cos x} dx \quad 2) \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx \quad 3) \int \sqrt{1+\sin 2x} dx \quad 4) \int \sqrt{1+\cos 2x} dx \quad 5) \int \sin^2 x dx$$

$$6) \int \cos^2 x dx \quad 7) \int \sin^2 \left( \frac{x}{2} \right) dx \quad 8) \int \frac{1-\tan x}{1+\tan x} dx \quad 9) \int \frac{1-\cos 2x}{1+\cos 2x} dx \quad 10) \int \sin^3 x dx \quad 11) \int \cos^3 x dx$$

$$12) \int \sin 3x \cdot \cos 7x dx \quad 13) \int \cos 5x \cdot \sin 3x dx \quad 14) \int \cos 8x \cos 2x dx \quad 15) \int \cos 3x \cos 4x dx$$

$$16) \int \frac{1+\tan^2 x}{1-\tan^2 x} dx \quad 17) \int \frac{1}{1+\cos 2x} dx.$$

### -Substitution Method-

$$\text{Evaluate: } 1) \int \frac{1}{x \cos^2(\log x)} dx. \quad 2) \int \frac{1}{x \log x} dx. \quad 3) \int e^{\sin x} \cos x dx. \quad 4) \int e^{\tan x} \cdot \sec^2 x dx.$$

$$5) \int \frac{\cos(\log x)}{x} dx. \quad 6) \int \frac{\sin(\log x)}{x} dx. \quad 7) \int x^{n-1} \cos(x^n) dx. \quad 8) \int \frac{\cos x - \sin x}{\cos x + \sin x} dx.$$

$$9) \int \frac{\sec^2 x}{3+\tan x} dx. \quad 10) \int \frac{x^2}{\sqrt{x+1}} dx. \quad 11) \int x\sqrt{1-2x} dx. \quad 12) \int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx.$$

$$13) \int e^{e^x} \cdot e^x dx. \quad 15) \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx. \quad 16) \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx \quad 17) \int \operatorname{cosec} x \cdot \log \left( \tan \frac{x}{2} \right) dx$$

$$18) \int \sin^2 x \cdot \cos x dx. \quad 19) \int \sin^3 x \cdot \cos x dx. \quad 20) \int \frac{e^x(x+1)}{\cos^2(xe^x)} dx. \quad 21) \int \frac{e^x(x+1)}{\sin^2(xe^x)} dx.$$

$$22) \int \frac{e^x(x-1)}{x^2 \cos^2 \left( \frac{e^x}{x} \right)} dx. \quad 23) \int \frac{\cot x}{\log(\sin x)} dx. \quad 24) \int \frac{3^{\tan^{-1} x}}{1+x^2} dx.$$

### -Integration of form $\int \frac{dx}{ax^2+bx+c}$ , $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ -

$$\text{Evaluate: } 1) \int \frac{dx}{x^2+4x+25} \quad 2) \int \frac{dx}{x^2+4x+5} \quad 3) \int \frac{dx}{x^2+3x+2} \quad 4) \int \frac{dx}{3+2x-x^2} \quad 5) \int \frac{dx}{2x^2+3x+2} \quad 6) \int \frac{dx}{\sqrt{13-6x-x^2}}$$

$$7) \int \frac{dx}{\sqrt{3-x-x^2}} \quad 8) \int \frac{dx}{\sqrt{7+5x-3x^2}} \quad 9) \int \frac{\cos x}{\sin^2 x + 10 \sin x + 26} dx \quad 10) \int \frac{dx}{\sqrt{x^2+4x+13}}.$$

### -Integration of form $\int \frac{dx}{a \pm b \sin 2x}$ , $\int \frac{dx}{a \pm b \cos 2x}$ , $\int \frac{dx}{a \sin 2x \pm b \cos 2x \pm c}$ , $\int \frac{dx}{a \pm b \sin x}$ , $\int \frac{dx}{a \pm b \cos x}$ -

$$\text{put } \sin 2x = \frac{2t}{1+t^2}, \cos 2x = \frac{1-t^2}{1+t^2}, dx = \frac{dt}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$$

$$\text{Evaluate: } 1) \int \frac{dx}{5+4\cos x} \quad 2) \int \frac{dx}{4-5\cos x} \quad 3) \int \frac{dx}{3+2\sin x} \quad 4) \int \frac{1}{5+3\cos 2x} dx \quad 5) \int \frac{dx}{3 \sin 2x + 2 \cos 2x} \quad 6) \int \frac{1}{4+5\cos 2x} dx$$

$$7) \int \frac{1}{5-3\sin 2x} dx \quad 8) \int \frac{dx}{3-2\cos x}$$

**-Integration of form**  $\int \frac{dx}{a \pm b \sin^2 x}$  ,  $\int \frac{dx}{a \pm b \cos^2 x}$  ,  $\int \frac{dx}{a \sin^2 x \pm b \cos^2 x \pm c}$  -

-Divide N & D by  $\cos^2 x$  , substitute  $\sec^2 x = 1 + \tan^2 x$  *only in D*, put  $x = t$  in D.

**Evaluate:** 1)  $\int \frac{dx}{3-2\sin^2 x}$  2)  $\int \frac{dx}{4\sin^2 x + 5\cos^2 x}$  3)  $\int \frac{dx}{4\cos^2 x + 9\sin^2 x}$  4)  $\int \frac{dx}{3+2\cos^2 x}$  5)  $\int \frac{dx}{2+3\sin^2 x}$

**-Integration by partial fraction-**

1)  $\int \frac{1}{(x+1)(2x+1)} dx$  2)  $\int \frac{dx}{x(x+1)}$  3)  $\int \frac{1}{(x+1)(x+2)}$  4)  $\int \frac{1}{2x^2-5x+2}$  5)  $\int \frac{x+1}{(x+2)(x+3)}$  6)  $\int \frac{x}{(x+1)(x-2)}$  dx

7)  $\int \frac{2x+1}{(x+1)(2x-1)}$  dx 8)  $\int \frac{x}{x^2+3x-4}$  dx 9)  $\int \frac{1}{x^2+5x+6}$  dx 10)  $\int \frac{1}{(x+2)(x+3)}$  dx 11)  $\int \frac{1}{(x+1)(x+2)(x+3)}$  dx

12)  $\int \frac{3x+1}{x^2+4x+3}$  dx 13)  $\int \frac{1-2x}{x^2-6x+9}$  dx 14)  $\int \frac{x^2+x+3}{(x+2)(x^2+1)}$  dx 15)  $\int \frac{1}{(x^2+4)(x+1)}$  dx 16)  $\int \frac{(x^2+1)}{(x+1)(x+2)(x+3)}$  dx

17)  $\int \frac{x+1}{x^3-4x}$  dx 18)  $\int \frac{x^2+6x-8}{x^3-4x}$  dx 19)  $\int \frac{x+1}{x^2(x-2)}$  dx 20)  $\int \frac{(2x^2+5)}{(x-1)(x+2)(x+3)}$  dx 21)  $\int \frac{x^2-1}{(x^2+1)(x^2+3)}$  dx

22)  $\int \frac{x^2}{(x^2+1)(x^2+3)}$  dx 23)  $\int \frac{x}{(x^2+4)(x^2+9)}$  dx 24)  $\int \frac{dx}{x(1+\log x)(2+\log x)}$  25)  $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}$  dx

26)  $\int \frac{\cos x}{(2+\sin x)(3+\sin x)}$  dx 27)  $\int \frac{e^x}{(e^x-1)(e^x+1)}$  dx 28)  $\int \frac{\log x}{x(2+\log x)(3+\log x)}$  dx 29)  $\int \frac{1}{x(2-\log x)(2\log x-1)}$  dx

30)  $\int \frac{dx}{\cos^2 x(1-\tan x)(2+\tan x)}$  31)  $\int \frac{2x+1}{(x+1)(x+2)(x+3)}$  dx .

**-Integration by parts-**  $\int u \cdot v dx = u \int v dx - \int \left[ v dx \cdot \frac{du}{dx} \right] dx$

**Evaluate:** 1)  $\int x^2 \log x dx$  2)  $\int x e^x dx$  3)  $\int x^2 e^x dx$  4)  $\int x \log x dx$  5)  $\int x \cos x dx$  6)  $\int x \sin x dx$

7)  $\int x \sin 2x dx$  8)  $\int x e^{2x} dx$  9)  $\int x \tan^{-1} x dx$  10)  $\int x \sec^{-1} x dx$  11)  $\int x \sec^2 x dx$  12)  $\int \frac{x dx}{1+\cos 2x}$

13)  $\int \frac{x^2 \tan^{-1} x}{1+x^2} dx$  14)  $\int \log x dx$  15)  $\int e^x \sin x dx$  16)  $\int e^x \sin 4x dx$  17)  $\int e^{2x} \sin 3x dx$

18)  $\int e^{2x} \cos 3x dx$  19)  $\int x^2 e^{3x} dx$  20)  $\int \sin^{-1} x dx$  21)  $\int \cot^{-1} x dx$  22)  $\int \tan^{-1} x dx$

23)  $\int \tan^{-1}(2x) dx$  24)  $\int x \sin^{-1} x dx$  25)  $\int x^2 \tan^{-1} x dx$  26)  $\int x \cdot \log(1+x) dx$  27)  $\int x^{2007} \log x dx$

28)  $\int \sin(\log x) dx$  29)  $\int \cos(\log x) dx$  30)  $\int \frac{x \tan^{-1} x dx}{(1+x^2)^{\frac{3}{2}}}$  31)  $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$  .

### TOPIC: Definite Integration

**Evaluate:** 1)  $\int_1^3 x^2 dx$  2)  $\int_4^9 \frac{dx}{x^2}$  3)  $\int_2^3 \frac{dx}{(x+1)}$  4)  $\int_1^2 \frac{dx}{(3x-2)}$  5)  $\int_0^2 \frac{dx}{(2x+1)}$  6)  $\int_2^{11} \frac{dx}{2x+11}$

7)  $\int_0^\infty e^{-x} dx$  8)  $\int_{-1}^1 \frac{1}{(1+x^2)} dx$  9)  $\int_0^{\log 2} e^{2x} dx$  10)  $\int_0^2 e^{2x} dx$  11)  $\int_0^1 \frac{2x dx}{x^2+4}$  12)  $\int_0^{\frac{\pi}{2}} \sin x \cos x dx$

13)  $\int_0^{\frac{\pi}{2}} \sin 5x \cdot \cos 3x \, dx$  14)  $\int_0^2 \frac{dx}{x^2-2x+2}$  15)  $\int_0^2 \frac{dx}{3x^2+2x+2}$  16)  $\int_0^4 \frac{dx}{\sqrt{4x-x^2}}$  17)  $\int_0^1 x \cdot e^x \, dx$   
 18)  $\int_0^{\frac{\pi}{4}} x \cdot \sec^2 x \, dx$  19)  $\int_1^e \log x \, dx$  20)  $\int_0^1 x^3 \cdot e^{x^2} \, dx$  21)  $\int_0^1 x \sin^{-1} x \, dx$   
 22)  $\int_0^1 x \cdot \tan^{-1} x \, dx$  23)  $\int_0^1 x^3 \cdot \tan^{-1} x \, dx$  24)  $\int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \cos x \, dx$  25)  $\int_0^{\frac{\pi}{5+4 \cos x}} \frac{dx}{5+4 \cos x}$  26)  $\int_0^{\frac{\pi}{4+5 \cos x}} \frac{dx}{4+5 \cos x}$   
 27)  $\int_0^{\frac{\pi}{2}} \sin^3 x \cdot \cos x \, dx$  .

Properties of Def. Integn. =  $\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$ ,  $\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$  ,  
 $\int_{-a}^a f(x) \, dx = \begin{cases} \int_0^a f(x) \, dx & , \text{if function is even.} \\ 0 & , \text{if function is odd.} \end{cases}$

Evaluate: 1)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin^2 x} \, dx}{\sqrt{\sin^2 x + \sqrt{\cos^2 x}}}$  2)  $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x} \, dx$  3)  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}$  4)  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\cot x}}$  5)  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt[3]{\cot x}}$   
 6)  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt[n]{\cot x}}$  7)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt[n]{\cot x}}$  8)  $\int_0^a \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} \, dx$  9)  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \, dx$  10)  $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}} \, dx$ .  
 11)  $\int_1^4 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} \, dx$  12)  $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} \, dx$  13)  $\int_0^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}} \, dx$  14)  $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \, dx$  15)  $\int_3^5 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}} \, dx$   
 16)  $\int_1^3 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}} \, dx$  17)  $\int_4^5 \frac{\sqrt{5-x}}{\sqrt{x-4}+\sqrt{5-x}} \, dx$  18)  $\int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}} \, dx$  19)  $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}} \, dx$   
 20)  $\int_2^{11} \frac{\sqrt{13-x}}{\sqrt{x}+\sqrt{13-x}} \, dx$  21)  $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} \, dx$  22)  $\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}} \, dx$  23)  $\int_0^{\frac{\pi}{2}} \log(\tan x) \, dx$   
 24)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\sin x + \cos x} \, dx$  25)  $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} \, dx$  26)  $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) \, d\theta$  27)  $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1+\tan x} \, dx$   
 28)  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} \, dx$  29)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\tan x}$  .

Area under the curve:

- 1) Find the area under the curve  $y = x^2$  from  $x = 0$  , to  $x = 3$  with  $x - axis$ .
- 2) Find the area bounded by curve  $y = x^3$  from  $x = 1$  , to  $x = 3$  with  $x - axis$  .
- 3) Find the area contained by curve  $y = 1 + x^3 + 2\sin x$  from  $x = 0$  to  $x = \pi$  .
- 4) Find the area enclosed by curve  $y = 3x^2$  and line  $x = 1$  , to  $x = 3$  and  $x - axis$  .
- 5) Find the area between lines  $y = 2x$  ,  $x - axis$  and ordinates at  $x = 1$  to  $x = 3$ .
- 6) Find the area enclosed between curve  $y = 3x - 2 - x^2$  and  $x - axis$  .
- 7) Find the area bounded by curves  $y = 4 - x^2$  and  $x - axis$  .
- 8) Find the area of region bounded by parabola  $y = 4x - x^2$  and  $x - axis$ .
- 9) Using integration find the area of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  .
- 10) Using integration find the area of ellipse  $\frac{x^2}{9} + \frac{y^2}{16} = 1$  .
- 11) Using integration find the area of circle  $x^2 + y^2 = a^2$  .
- 12) Using integration find the area of circle  $x^2 + y^2 = 16$  .
- 13) Using integration find the area of circle  $x^2 + y^2 = 9$
- 14) Using integration find the area of circle  $x^2 + y^2 = 25$
- 15) Using integration find the area of circle  $x^2 + y^2 = 36$  .

Area between two curve :

- 1) Calculate the area enclosed by  $y^2 = 8x$  and line  $x = 2$ .
- 2) Find the area enclosed by  $y^2 = x$  and line  $y = x$ .
- 3) Find the area enclosed by curve  $y = x^2$  and line  $y = x$ .
- 4) Find area between parabola  $y^2 = 8x$  and the straight line  $y = 2x$ .
- 5) Find area between parabola  $y = x^2$  and the straight line  $y = 4x - 3$ .
- 6) Determine area enclosed by curve  $y^2 = 4x$  and  $y = x$ .
- 7) Find the area enclosed by  $y^2 = 4x$  and  $2x - y = 4$ .
- 8) Find area between parabola  $y = x^2 + 3$  and the line  $y = x + 3$ .
- 9) Find area between parabola  $y = x^2 + 1$  and the line  $y = 2x + 1$ .
- 10) Find the area enclosed by parabola  $y^2 = x$  and  $x^2 = y$ .
- 11) Find the area enclosed by parabola  $y^2 = 2x$  and  $x^2 = 2y$ .
- 12) Find the area between parabola  $y^2 = 4x$  and  $x^2 = 4y$ .
- 13) Find the area bounded by parabola  $y^2 = 9x$  and  $x^2 = 9y$ .
- 14) Find the area bounded by parabola  $y^2 = 2x$  and  $x - y = 4$ .

**TOPIC: Differential Equation (D.E)**

1) Define order and degree of D.E.

2) Find order and degree of following D.E's:

$$1) \left(\frac{d^3y}{dx^3}\right)^3 + 2\left(\frac{d^2y}{dx^2}\right)^4 + 5\frac{dy}{dx} + 6y = 4 \quad 2) \frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 - 6y = 0$$

$$3) 2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0 \quad 4) \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 \quad 5) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 5\frac{d^2y}{dx^2}.$$

$$6) \frac{d^2y}{dx^2} + \sqrt{1 + \frac{dy}{dx}} = 0 \quad 7) \sqrt{\frac{d^3y}{dx^3}} + \frac{dy}{dx} = y \quad 8) \frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}.$$

$$9) \frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3} \quad 10) \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \quad 11) \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$

$$12) \frac{d^3y}{dx^3} = \left[ k + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} \quad 13) \left[ 2 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = k \frac{d^2y}{dx^2} \quad 14) \frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = y.$$

$$15) \sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}} \quad 16) \sqrt[3]{\frac{d^2y}{dx^2} + 4x} = \sqrt{\frac{dy}{dx} - 1} \quad 17) \sqrt[3]{\frac{dy}{dx} + y} = \sqrt[4]{\frac{d^2y}{dx^2}}$$

$$18) \left( \frac{d^3y}{dx^3} \right)^{\frac{1}{2}} + \left( \frac{dy}{dx} \right)^{\frac{1}{3}} = 3 \quad 19) y \left( \frac{dy}{dx} \right)^3 = \frac{x \frac{dy}{dx} + 5}{\frac{dy}{dx}} \quad 20) \frac{y-x}{\frac{dy}{dx}} = \left( \frac{dy}{dx} \right)^2.$$

3) Form the differential equation of  $y = Ae^{3x}$ .

4) Form the differential equation by eliminating constant from  $y = A \sin x + B \cos x$ .

5) Form a differential equation if  $y = A \cos 3x + B \sin 3x$ .

6) Form a differential equation if  $y = A \cos(\log x) + B \sin(\log x)$ .

7) Form the differential equation of  $y = ax^2$ .

8) Form a differential equation if  $y = Ae^{2x} + Be^{-2x}$ .

9) Form the differential equation of  $y = e^{m \tan^{-1} x}$ .

10) Form the differential equation of  $y^2 = 4ax$ .

11) Verify that  $y = e^{-x}$  is a solution of  $\frac{d^2y}{dx^2} - y = 0$ .

12) Show that  $y = e^{ax}$  is a general solution of  $x \frac{dy}{dx} = y \log y$ .

13) Verify that  $y = \cos x$  is a solution of  $\frac{d^2y}{dx^2} + y = 0$ .

14) Verify that  $y = \log x$  is a solution of  $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ .

15) Verify that  $y = \sin(\log x)$  is a solution of  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

16) Verify that  $y = e^{m \sin^{-1} x}$  is a solution of  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$ .

17) Show that  $y^2 = ax^2$  is a solution of  $x \left( \frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} + ax = 0$ .

### Variable Separable Form :-

1) Solve :  $x dy - y dx = 0$ .

2) Solve :  $x^2 dx = y^2 dy$ .

3) Solve the differential equation  $\frac{1}{y^2} dx = \frac{1}{x} dy$ .

4) Solve :  $e^y \frac{dy}{dx} = x^2$ .

5) Solve the differential equation  $\sqrt{1 - y^2} dx - \sqrt{1 - x^2} dy = 0$ .

6) Solve the differential equation  $2x \cos y dx - (1 + x^2) \sin y dy = 0$ .

7) Solve the differential equation  $x(1 + y^2) dx + y(1 + x^2) dy = 0$ .

8) Find particular solution of differential equation  $(1 + x^3) dy - x^2 y dx = 0$ , at  $x = 1, y = 2$ .

9) Solve the differential equation  $(1 + x^2) dy - (1 + y^2) dx = 0$ .

10) Solve :  $\sec^2 x \cdot \tan y dx + \sec^2 y \cdot \tan x dy = 0$ .

11) Solve the differential equation  $\frac{dy}{dx} = e^{(x-y)} \cdot x^2$ .

12) Solve the differential equation  $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ .

13) Solve the differential equation  $\frac{dy}{dx} = e^{3x-2y} + x^2 \cdot e^{-2y}$  .

14) Solve :  $\frac{dy}{dx} = e^{x-y} + x^2 \cdot e^{-y}$  .

15) Solve :  $\frac{dy}{dx} = e^{x-y} + x \cdot e^{-y}$

16) Solve :  $\cos x \cdot \cos y dx - \sin x \cdot \sin y dy = 0$

17) Solve :  $(x + 1) dy + (y + 1) dx = 0$

18) Solve :  $\frac{dy}{dx} = xy + x + y + 1$  .

**-Reducible to variable separable form \_**

1) Solve the differential equation  $(x + y + 1) \frac{dy}{dx} = 1$  .

2) Solve :  $\frac{dy}{dx} = (4x + y + 1)^2$

3) Solve :  $\frac{dy}{dx} = \sin(x + y)$

4) Solve :  $\frac{dy}{dx} = \cos(x - y)$

5) Solve :  $\frac{dy}{dx} = \cos(x + y)$

6) Solve :  $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$

7) Solve :  $(4x + y)^2 \frac{dy}{dx} = 1$

8) Solve :  $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

**Linear diff. equation –**

1) Find integrating factor of following equations:

1)  $\frac{dy}{dx} + y \tan x = \cos^2 x$  2)  $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$  3)  $\frac{dy}{dx} + y \tan x = \sin 2x$

4)  $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$  .

**Solve:** 1)  $\frac{dy}{dx} + \frac{y}{x} = x^3$  . 2)  $x \frac{dy}{dx} + y = x^3$  3)  $x \frac{dy}{dx} - y = x^2$  4)  $\frac{dy}{dx} + (\cot x) y = \cos x$ .

5)  $x \frac{dy}{dx} - y = x^2 \cos x$  6)  $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$  . 7)  $\sin x \frac{dy}{dx} + y \cos x = x \sin x$  .

8)  $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$  9)  $x \frac{dy}{dx} + y = x^2 e^x$  10)  $(x + 1) \frac{dy}{dx} - y = e^x (x + 1)^2$

11)  $(1 + x) \frac{dy}{dx} - y = e^{3x} (1 + x)^2$  12)  $\frac{dy}{dx} + y \tan x = \sec x$  13)  $\cos^2 x \frac{dy}{dx} + y = \tan x$  .

14)  $x \log x \frac{dy}{dx} + y = 2 \log x$  . 15)  $(1 + y^2) dx = (\tan^{-1} y - x) dy$  .

**TOPIC:            PROBABILITY(Only for ME)**

**Binomial Distribution :-**

- 1) An unbiased coin is tossed 5 times .Find the probability of getting a head .
- 2) An unbiased coin is tossed 6 times .Find the probability of getting i) 2 heads ii) exactly 4 heads .
- 3) An unbiased coin is tossed 5 times .Find the probability of getting i) 3 heads ii) atleast 4 heads .
- 4) On an average 10% of the products manufactured by a machine are defective . if from these products four are chosen at random , find the probability that one of them is defective .
- 5) If the chance that out of 10 telephone lines one of the line is busy at any instant is 0.2. what is the chance that 5 of the lines are busy .
- 6) Assuming that 2 in 10 industrial accidents are due to fatigue. Find the probability that exactly 2 out of 8 accident will be due to fatigue .
- 7) The probability that man aged 60 will live to be 70 is 0.65 . what is the probability that out of 10 men which are now 60 at least 7 would live to be 70 .
- 8) On an average 3 of 10 electric components in a packet are defective . if 4 items are selected at random and tested , what is the probability that not more than one defective .
- 9) 10 percent of the component manufactured by a company are defective . if twelve components selected at random ,find the probability that atleast two will be defective .
- 10) In 200 sets of tosses of 5 fair coins in how many ways you can expect i) at least two heads ii)at the most two heads .
- 11) If 10% of bolts produced by a machine are defective .Determine the probability that out of 10 bolts chosen at random i) one ii) none iii) at most 2 bolts will be defective .
- 12) If 20% of bolts produced by a machine are defective .Determine the probability that out of 4 bolts drawn i) one is defective ii) At most two are defective .

**-Poisson Distribution**

- 1) Fit a Poisson distribution to set of observations :

$x_i$	1	2	3	4	5
$f_i$	122	60	15	02	01

- 2) Fit a Poisson distribution to set of observations :

$x_i$	20	30	40	50	60	70
$f_i$	8	12	20	10	6	4

- 3) If a random variable has a poisson distribution such that  $P(2) = P(3)$  . Find  $P(5)$  .
- 4) If a random variable has a poisson distribution such that  $P(3) = P(4)$  . Find  $P(1)$  .
- 5) A firm produces articles of which 0.1 percent are defectives out of 500 articles . If wholesaler purchases 100 such cases , how many can be expected to have one defective .( $e^{-0.5} = 0.6065$ ) .
- 6) The probability of getting an item defective is 0.005 . What is the probability that exactly 3 items in a sample of 200 are defective . ( $e^{-1} = 0.3679$ ) .

7) Using poisson distribution , find the probability that king of diamond will be drawn from the pack of well shuffled cards atleast once in 156 consecutive trial . . (given :  $e^3 = 20$ )

8) Using poisson distribution , find the probability that queen of club will be drawn from the pack of well shuffled cards atleast once in 156 consecutive trial . . (given :  $e^3 = 20$ ) .

9) Assuming that the probability of a fatal accident in a factory during the year is  $\frac{1}{1200}$  . calculate the probability that in a factory employing 300 workers there will be atleast two fatal accidents in a year .  
(given :  $e^{-0.25} = 0.7788$ )

10) Assuming that 2 in 10 industrial accidents are due to fatigue . find the probability exactly 2 out of 8 accidents will be due to fatigue by using poisson distribution .

11) If 2% of electric bulbs manufactured by a company are defective ,find the probability that in a sample of 100 bulbs i) 3 bulbs will be defective ii) at most 2 bulbs will be defective .

12) If 3% of electric bulbs manufactured by a company are defective ,find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective . (given :  $e^{-3} = 0.04974$ )

13) If 20% of bolts produced by a machine are defective .Determine the probability that out of 4 bolts drawn i) one is defective ii) At most two are defective .

14) if the probability of a bad reaction from a certain injection is 0.001 , determine the chance that out of 2000 individuals more than two will get a bad reaction . (Given :  $e^2 = 7.3891$ )

15) The number of road accidents met with taxi drivers follows poisson distribution with mean 2 . out of 5000 taxi in the city , find the number of drivers i) who does not meet with an accidents ii) who met with an accident more than 3 times . (Given :  $e^{-2} = 0.1353$ ) .

### Normal Distribution –

1) The life time of certain kind of electronic devices have a mean of 300 hrs and S.D of 25 hrs. Find the probability that any one of these electronic devices will have a life time of more than 350 hrs.

(Area between  $Z=0$  to  $Z=2$  is 0.4772) .

2) The average life of bulb is 120 days with a S.D 20 days ,1000 bulbs are purchased in a factory .How many bulbs will expire in less than 90 days . (Area between  $Z=0$  to  $Z=1.5$  is 0.4332) .

3) In a sample of 1000 students the mean of certain test is 14 and S.D is 2.5 assuming the distribution to be normal .Find how many score above 18 . (Area between  $Z=0$  and 1.6 is 0.4452) .

4) In a test on 2000 electric bulbs, it was found that the life of particular make was normally distributed with average life of 2040 hrs. and standard deviation of 60 hrs. Estimate the no. of bulbs likely to burn for: i) between 1920 hrs and 2160 hrs. ii) More than 2150 hrs. (Area between  $Z=0$  to  $Z=2$  is 0.4772) .

5) In a certain examination 500 students appeared, mean score is 68 and S.D is 8. Find the number of students scoring i) Less than 50 ii) more than 60 . (Area between  $Z=0$  to  $Z=1$  is 0.3413).

6) In a sample of 1000 cases , the mean of certain test is 14 and S.D is 2.5. Assuming the distribution to be normal find a) How many students score between 12 and 15 b) How many students score above 18 . ( given : Area between  $A(0.8)=0.2881$  ,  $A(0.4)=0.1554$  ,  $A(1.6)=0.4452$ ) .

- 7) In a sample of 100 dry battery cells tested to find the length of life produced the following results mean = 12 hrs. and S.D is 3 hrs .Assuming that the data are normally distributed ,what percentage of battery cells are expected to have life , a) more than 15 hours b)less than 6 hours c) between 10 and 15 hours .

**TOPIC: NUMERICAL METHOD'S (Only for CO)**

- 1) Using Bisection method find the approximate root of following equation's-
- $x^2 - 2x - 5 = 0$  upto three iterations.
  - $x^2 - \log_e x = 1.8$  upto three iterations.
  - $x^3 - 5x - 3 = 0$  root lies between 2 & 3. upto three iterations.
  - $x^3 - 9x - 5 = 0$  in the interval(3,4).
  - $\sqrt{40}$  . carry out three iterations.
  - Find first two roots of equation  $x^3 - 2x - 5 = 0$ .
- 2) Using Regula Falsi / False position method find the approximate root of following equations-
- $x^2 - 2x - 9 = 0$  up to three iterations.
  - $x^3 + 2x - 50 = 0$
  - $xe^x - 8 = 0$
  - $x^3 + 3x - 20 = 0$  between 2 & 3 upto three iterations.
  - $x - \log x = 6$
  - $x^3 - 9x + 1 = 0$  .
- 3) By using Newton Raphson method find the root of following equations upto three iteration—
- $x^4 - x - 5 = 0$
  - $x^3 - x - 1 = 0$  by taking initial root 1 .
  - $\sqrt[3]{100}$
  - $x^3 - 20 = 0$
  - $\sqrt{10}$
  - $x^3 - 4x + 1 = 0$  in (1,2) .
- 4) Using Gauss elimination method solve the following equations—
- $2x + 5y - 3z = 22, x - 2y + 4z = 3, 6x - 7y + z = 10$ .
  - $x + y - z = 3, x - y + 4z = 4, 2x + y + z = 7$ .
  - $3x + 4y + z = -3, x + 2y + z = -1, 2x - y + z = 6$  .

d)  $3x - 4y - 6z = 4, 3x + 12y + 9z = -2, 9x + 8y + 3z = 3.$

5) Solve the following equations using Jacobi's method

a)  $10x + y + 2z = 9, -2x + 10y + 3z = 5, 3x - y + 10z = -8.$

b)  $20x - 3y + z = 17.5, 4x + 15y - 5z = 16.5, x - 7y + 10z = 10.5.$

c)  $9x - 5y + z = 30, 3x + 10y + 2z + 7 = 0, 6x + 5y + 12z - 18 = 0.$

d)  $30x + 10y - 3z = 96, 4x + 25y + 2z = 53, 2x + 5y + 20z = -6.$

e)  $12x + 4y + 3z = 19, 5x + 12y + 2z = 19, 7x + y + 12z = 20.$

6) Solve using Gauss Seidal method following equations----

a)  $10x + 3y + 2z = 2, 5x + 10y + z = 18, 7x + 2y + 10z = 27.$

b)  $-15x + 9y + 4z = 2, 6x - 14y + 3z = 5, 2x + 5y - 13z = 6.$

c)  $x + 7y - 3z = -22, 5x - 2y + 3z = 18, 2x - y + 6z = 22$

d)  $5x - y = 9, x - 5y + z = -4, y - 5z = 6$  using initial approximations  $x_0 = 1.5,$   
 $y_0 = 0.5, z_0 = -0.5.$

e)  $8x + y + z = 10, 3x - 8y + 5z = 0, 7x + 8z = 15,$  taking initial approximations as  
 $x_0 = y_0 = z_0 = 0.75.$

**TOPIC: NUMERICAL INTEGRAL (Only for CE)**

**Trapezoidal Rule-**

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

1) Evaluate using Trapezoidal rule

i)  $\int_2^7 \frac{dx}{x}$  taking  $n=5$

ii)  $\int_0^1 \frac{dx}{1+x^2}$  taking  $n=4$

2) Evaluate  $\int_0^{\frac{\pi}{2}} \cos x dx$  using trapezoidal rule by dividing the interval  $[0, \frac{\pi}{2}]$  into three equal parts

3) Evaluate using Trapezoidal rule  $\int_{-1}^1 (1 + x + x^2 + x^3) dx$  by taking  $n=2$

4) Find  $\int_0^{\frac{\pi}{4}} f(x) dx$  using Trapezoidal rule

Given

X	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
tanx	0	0.4141	1

5) Evaluate  $\int_{-3}^3 x^4 dx$  using trapezoidal rule by dividing the interval  $[-3,3]$  into seven equal parts

6) Evaluate using Trapezoidal rule

i)  $\int_0^1 x^2 dx$  taking  $h=0.2$

ii)  $\int_0^1 \frac{dx}{1+x}$  taking  $n=2$

7) Compute  $\int_0^2 (1+x^3) dx$  using trapezoidal rule by dividing the interval  $[0,2]$  into four equal sub intervals

8) Find  $\int_0^8 f(x) dx$  using Trapezoidal rule

Given

X	0	2	4	6	8
F(x)	2	5	10	17	26

9) Find  $\int_3^8 \log_e x dx$  using Trapezoidal rule

X	3	4	5	6	7	8
$\log_e x$	1.0986	1.363	1.6094	1.7918	1.9459	2.0794

### Simpson's one third rule-

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \dots)]$$

1) Evaluate  $\int_0^2 e^{-x} dx$  using Simpson's one third rule given that,

X	0	1/2	1	3/2	2
$e^{-x}$	1	0.6064	0.3676	0.2231	0.1353

2) Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  using Simpson's one third rule taking  $h=1/4$

3) Evaluate  $\int_1^5 \frac{dx}{x+2}$  using Simpson's one third rule dividing the interval  $[1,5]$  into four equal sub intervals

4) Evaluate  $\int_0^{\pi/2} \sqrt{\cos x} dx$  using using Simpson's one third rule by dividing into eight equal parts

5) Evaluate  $\int_1^3 \frac{dx}{x}$  using Simpson's one third rule taking  $h=0.5$

6) Evaluate  $\int_0^5 \frac{dx}{4x+5}$  using Simpson's one third rule taking  $n=10$

7) Evaluate  $\int_0^6 \frac{dx}{1+x^2}$  using Simpson's one third rule taking seven equidistant ordinates

8) Evaluate  $\int_0^{\pi} \sin x \, dx$  using Simpson's one third rule divide 4 equal sub intervals

9) Apply Simpson's one third rule to find  $\int_0^2 f(x) \, dx$  using following data

x	0	1.0	1.5	2.0
F(x)	1.1	2.4	5.7	8.1

10) By using Simpson's one third rule  $\int_0^2 \frac{dx}{1+x^3}$  with n=4

**Simpson's 3/8 rule-**

$$\int_{x_0}^{x_n} f(x) \, dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + y_9 \dots \dots)]$$

1) Evaluate  $\int_0^4 x^2 \, dx$  using Simpson's 3/8 rule taking  $h=0.5$

2) Evaluate  $\int_1^2 \frac{1}{x} \, dx$  using Simpson's 3/8 rule taking  $n=5$

3) Evaluate  $\int_0^4 x^2 \, dx$  using Simpson's 3/8 rule taking  $h=0.5$

4) Evaluate  $\int_0^1 \frac{dx}{1+x}$  using Simpson's 3/8 rule dividing the interval [0,1] into eight equal sub

5) Evaluate  $\int_1^7 f(x) \, dx$  using Simpson's 3/8 rule given that,

X	1	2	3	4	5	6	7
F(x)	2.105	2.808	3.614	4.604	5.857	7.451	9.467

6) Evaluate  $\int_0^6 f(x) \, dx$  using Simpson's 3/8 rule given that,

X	0	1	2	3	4	5	6
F(x)	0.146	0.161	0.176	0.190	0.204	0.217	0.230

7) Evaluate  $\int_0^{\pi} \sin x \, dx$  using Simpson's 3/8 rule taking n=10

8) Evaluate  $\int_0^{1.2} \log(1+x^2) \, dx$  using Simpson's 3/8 with h=0.2

9) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+x} \, dx$  using Simpson's 3/8 rule taking  $h=\frac{\pi}{12}$

10) Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  using Simpson's 3/8 rule taking  $h=1/6$