



SARASWATI Education Society's
SARASWATI Institute of Technology

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Kharghar, Navi Mumbai - 410 210.

**SARASWATI
INSTITUTE
OF TECHNOLOGY**

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MATHEMATICS FORMULAE

SEM -I, SEM- II, SEM-III

DIPLOMA

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- ❖ “**Mathematics** is king of art; queen of sciences.”
- ❖ “If you fail never give up because **F.A.I.L** means “first attempt in learning”.
- ❖ “If people do not believe that **mathematics** is simple they do not realize how complicated life is.”
- ❖ “**Mathematics** is a great motivator for all humans because its career starts with zero but never end (infinity).”
- ❖ “Life is good for only two things; discovering **mathematics** and teaching **mathematics**”
- ❖ “**Mathematics** has strong heart! So only it will face many problems!!”
- ❖ “**Mathematics** is the brain’s sharpener.”
- ❖ “For the thing of this world cannot be made know without knowledge of **Mathematics.**”
- ❖ “**Mathematics** proficiency is the gateway to a number of incredible careers that student may never have consider.”

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Rules of(+, -, ×, ÷):

$$+(+) +=$$

$$-(+) -=$$

Add Numbers and
give sign of equality
to Answer

Ex

$$2+2=4$$

$$-2+ - 2= - 4$$

$$-(+) +=$$

$$+(+) -=$$

Subtract Numbers and
Give sign of greater
number to Ans.

$$-3+2=-1$$

$$2+(-3)= - 1$$

$$+(\times) +=$$

$$-(\times) -=$$

Multiply Numbers and
given +ve sign to
Answer

$$2 \times 2 =4$$

$$(- 2) \times (- 2) =4$$

$$+(\times) -=$$

$$-(\times) +=$$

Multiply Numbers and
give -ve sing to Answer

$$2 \times (- 2) = -4$$

$$- 3 \times 2 = - 6$$

$$+ \div +=$$

$$- \div -=$$

Divide and give +ve
sign to Ans

Ex

$$2 \div 2=1$$

$$(-2) \div (- 2)= 1$$

$$+ \div -=$$

$$- \div +=$$

Divide and give
- ve sign to Ans

Ex

$$2 \div (- 2)=-1$$

$$(-2) \div 2= -1$$

Type of Numbers:

N = Set of Natural Numbers

$$N = \{1,2,3,4,\dots\dots\dots\}$$

W = Set of whole Numbers

$$W = \{0,1,2,3,4,\dots\dots\dots\}$$

I = Set of integers

$$I = \{\dots\dots -3,-2,-1,0,1,2,3\dots\}$$

Q = Set of rational Numbers

$$Q = \{ p/q, p,q \in \mathbb{R} . q \neq 0 \}$$

$\overline{Q}$ = Set of irrationals

$$\overline{Q} = \{x \in \mathbb{R} / \text{but } x \notin Q\}$$

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R = Set of Real Numbers

$R = \{ x \in (Q \cup Q) \}$

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LAW OF INDICES:

$$\text{i) } a^m \times a^n = a^{m+n}$$

$$\text{ii) } a^m \div a^n = a^{m-n}$$

$$\text{vii) } e^{\infty} = \infty$$

$$\text{iii) } (a^m)^n = a^{mn}$$

$$\text{iv) } (a^{-n}) = \frac{1}{a^n}$$

$$\text{v) } a^0 = 1, a \neq 0$$

$$\text{vi) } (ab)^m = a^m b^m$$

RULES OF LOGARITHM:

If $a^x = b$ then $x = \log_a b$

$$\text{i) } \log 1 = 0 \text{ (any base)}$$

$$\text{ii) } \log_m a^n = n \log_m a$$

$$\text{iii) } \log_m a + \log_m b = \log_m ab$$

$$\text{iv) } \log_m a - \log_m b = \log_m \frac{a}{b}$$

$$\text{v) } \log_m m = 1, m > 0, m \neq 1$$

$$\text{vi) } \log_a b = \frac{\log_m a}{\log_m b} \quad a, b, m > 0$$

$$\text{vii) } \log_b a = \frac{1}{\log_a b}$$

$$\text{viii) } a^{\log_a f(x)} = f(x)$$

$$\text{ix) } a^{-\log_a f(x)} = \frac{1}{f(x)}$$

Factorization Formula:-

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

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SEM –I BASIC MATHEMATICS

Determinants: An arrangement of numbers in equal number of rows and columns between two vertical lines. Denoted by D or Δ

Eg.

$$D = \begin{vmatrix} 1 & 6 & 9 \\ 5 & 2 & -2 \\ 0 & 3 & 4 \end{vmatrix} = 1[(4*2)-(-2*3)] - 6[(5*4)-(-2*0)] + 9[(5*3)-(2*0)] = 29$$

Cramer's rule :-

Two unknowns: $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ then $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$ where

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

Three Unknowns: $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

then $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$ where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Matrix: Matrix is an arrangement of $m \times n$ numbers enclosed in a square bracket along with m rows and n columns.

eg :- $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ order is 2×2

Types of Matrices

1) **Row Matrix**: - A Matrix is having only one row.

Eg. [a b c]

2) **Column Matrix** :- A matrix having only one column

eg.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

3) **Square matrix** :- A Matrix having same number of rows and columns

(m = n)

eg. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 2x2

4) **Diagonal Matrix**: A square matrix whose all non diagonal elements are zero.

eg. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ 3x3

5) **Scalar Matrix**: - A Diagonal matrix whose all diagonal element are equal.

eg. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 3x3

6) **Identity / Unit Matrix**: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 2x2 $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 3x3

7) **Symmetric matrix** = A square matrix in which $a_{ij} = a_{ji}$, $i \neq j$

eg.
$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

8) **Skew symmetric matrix**: A square matrix in which $a_{ij} = -a_{ji}$, $i \neq j$ and diagonal elements are zero .

eg.
$$\begin{pmatrix} 0 & -b & -c \\ b & 0 & e \\ c & -e & 0 \end{pmatrix}$$

9) A square matrix 'A' is singular if $|A| = 0$
 is Nonsingular if $|A| \neq 0$

10) Transpose of square matrix is obtained by interchanging rows and columns. It is denoted by A^T or A^1

- $(A+B)^T = (A)^T + (B)^T$
- $(AB)^T = (B)^T(A)^T$
- $(A^T)^T = (A)$

11) Adj. A is Transpose of co-factor matrix.

12) Inversion method: $X = A^{-1}B$

$$A^{-1} = \frac{1}{|A|} \text{adj } A, |A| \neq 0$$

Partial Fraction: Partial Fraction is method of rewriting the fractions into simple fraction.

Type I: D^r has distinct linear factors

$$\frac{ps+q}{(as \pm b)(cs \pm d)} = \frac{A}{(as \pm b)} + \frac{B}{(cs \pm d)}, \text{ where A and B are Constants to be determined.}$$

Type II: D^r has repeated factors:

$$\frac{ps+q}{(as \pm b)^n} = \frac{A}{(as \pm b)} + \frac{B}{(as \pm b)^2} + \frac{C}{(as \pm b)^3} + \dots$$

Type III : D^r has quadratic irreducible factors

$$\frac{ps+q}{(a_1s^2\pm b_1s\pm c_1)(a_2s^2\pm b_2s\pm c_2)} = \frac{as+b}{(a_1s^2\pm b_1s\pm c_1)} + \frac{cs+d}{(a_2s^2\pm b_2s\pm c_2)}$$

-Proper fraction: $\deg N^r < \deg D^r$ -Improper fraction: $\deg N^r \geq \deg D^r$

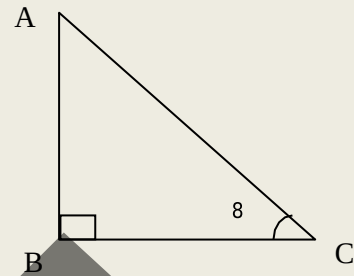
Trigonometry:

1) Trigonometric Ratios:

In ΔABC , $\angle B = 90^\circ$, $\angle A$ & $\angle C$ are acute angles.

$$\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{AB}{AC}, \quad \cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{AB}{BC}, \quad \cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{BC}{AB}$$





Trigonometric Table :-

Angles T.ratios	0°	30°	45°	60°	90°	180°
Sine	0	1/2	1/√2	√3/2	1	0
Cosine	1	√3/2	1/√2	1/2	0	-1
Tan	0	1/√3	1	√3	∞	0
Cot	∞	√3	1	1/√3	0	∞
Sec	1	2/√3	√2	2	∞	-1
Cosec	∞	2	√2	2/√3	1	∞

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Sign of Trigonometric Ratios :-

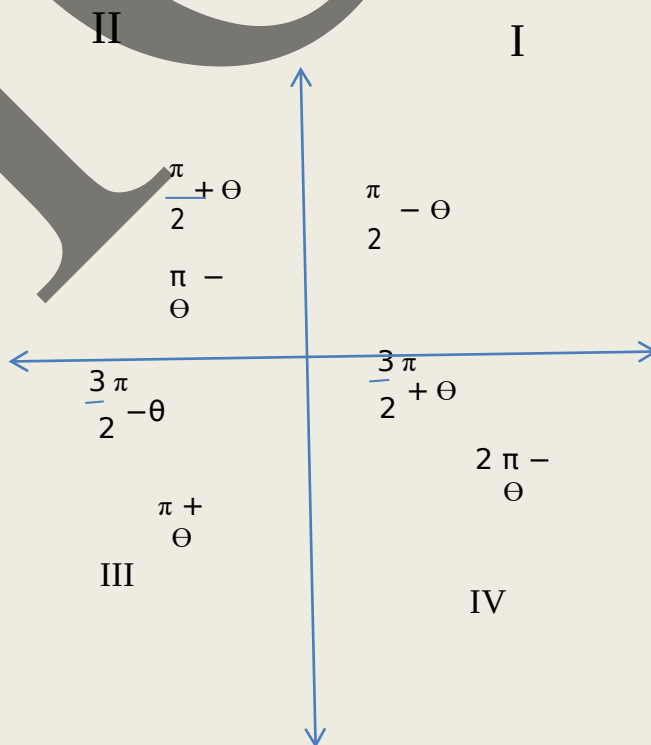
Quadrant +

I: All are +ve

II: Sin & Cosec are +ve

III: tan & Cot are +ve

IV: Cos & Sec are +ve





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Trigonometric Ratios of Negative angle :

$$\sin(-\theta) = -\sin \theta \quad \operatorname{Cosec}(-\theta) = -\operatorname{Cosec} \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

COMPOUND ANGLES:-

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan\left(\frac{n}{4} \pm \theta\right) = \frac{1 \pm \tan \theta}{1 \mp \tan \theta}$$

Transformation Angles :-

$$\sin(\pi/2 - \theta) = \cos \theta$$

$$\sin(\pi/2 + \theta) = +\cos \theta$$

$$\cos(\pi/2 - \theta) = \sin \theta$$

$$\cos(\pi/2 + \theta) = -\sin \theta$$

$$\tan(\pi/2 - \theta) = \cot \theta$$

$$\tan(\pi/2 + \theta) = -\cot \theta$$

$$\cot(\pi/2 - \theta) = \tan \theta$$

$$\cot(\pi/2 + \theta) = -\tan \theta$$



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$$\sec(\pi/2 - 8) = \operatorname{cosec} 8$$

$$\sec(\pi/2 - 8) = -\operatorname{cosec} 8$$

$$\operatorname{Cosec}(\pi/2 - 8) = \sec 8$$

$$\operatorname{Cosec}(\pi/2 + 8) = +\sec 8$$

STO



Non transformation Angles.

$$\sin(\pi - \theta) = \sin \theta$$

$$\cos(\pi - \theta) = -\cos \theta$$

$$\tan(\pi - \theta) = -\tan \theta$$

$$\cot(\pi - \theta) = -\cot \theta$$

$$\sec(\pi - \theta) = -\sec \theta$$

$$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec} \theta$$

$$\sin(2n\pi \pm \theta) = \pm \sin \theta$$

$$\sin(\pi + \theta) = -\sin \theta$$

$$\cos(\pi + \theta) = -\cos \theta$$

$$\tan(\pi + \theta) = \tan \theta$$

$$\cot(\pi + \theta) = \cot \theta$$

$$\sec(\pi + \theta) = -\sec \theta$$

$$\operatorname{cosec}(\pi + \theta) = -\operatorname{cosec} \theta$$

$$\cos(2n\pi \pm \theta) = \cos \theta$$

Trigonometric Ratios of

Multiple Angles

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

Submultiples angles

$$\sin \frac{\theta}{2} = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$= \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\cos^2 \frac{\theta}{2} = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

$$= 2 \cos^2 \frac{\theta}{2} - 1$$

$$= 1 - 2 \sin^2 \frac{\theta}{2}$$

$$= \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$



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$$1 + \cos 2\theta = 2 \cos^2 \theta$$

$$1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}$$

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$$1 - \cos 2\theta = 2 \sin^2 \theta$$

$$1 - \cos \theta = 2 \sin^2 \theta / 2$$

$$1 + \sin 2\theta = (\cos \theta + \sin \theta)^2$$

$$1 + \sin 2\theta = (\cos \theta / 2 + \sin \theta / 2)^2$$

$$1 - \sin 2\theta = (\cos \theta - \sin \theta)^2$$

$$1 - \sin 2\theta = (\cos \theta / 2 - \sin \theta / 2)^2$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

Factorization formula

$$\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$\sin \theta + \sin \theta = 2 \sin \theta \cos \theta$$

$$\sin \theta - \sin \theta = 2 \sin \theta \cos \theta$$

Defactorization formula

$$2 \sin A \cos B = \sin (A+B) + \sin (A-B)$$

$$2 \cos A \sin B = \sin (A+B) - \sin (A-B)$$

$$2 \cos A \cos B = \cos (A+B) + \cos (A-B)$$

$$-2 \sin A \sin B = \cos (A+B) - \cos (A-B)$$

$$\cos \theta + \cos \theta = 2 \cos \theta \sin \theta$$

$$\cos \theta - \cos \theta = -2 \sin \theta \sin \theta$$



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Inverse Trigonometry

If $x = \sin \theta$ then $\theta = \sin^{-1} x$

$$\sin^{-1}(\sin \theta) = \theta$$

$$\cos^{-1}(\cos \theta) = \theta$$

$$\tan^{-1}(\tan \theta) = \theta$$

$$\sin^{-1} x = \operatorname{Cosec}^{-1} \left(\frac{1}{x} \right)$$

$$\cos^{-1} x = \sec^{-1} \left(\frac{1}{x} \right)$$

$$\tan^{-1} x = \operatorname{Cot}^{-1} \left(\frac{1}{x} \right)$$

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$\sin^{-1} x + \cos^{-1} x = \pi/2$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \pi/2$$

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$= \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$\operatorname{Cot}^{-1}(\operatorname{Cot} \theta) = \theta$$

$$\sec^{-1}(\sec \theta) = \theta$$

$$\operatorname{Cosec}^{-1}(\operatorname{Cosec} \theta) = \theta$$

$$\operatorname{cosec}^{-1} x = \sin^{-1} \left(\frac{1}{x} \right)$$

$$\sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right)$$

$$\operatorname{cot}^{-1} x = \tan^{-1} \left(\frac{1}{x} \right)$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\operatorname{cot}^{-1}(-x) = \pi - \operatorname{cot}^{-1} x$$

$$\tan^{-1} x + \operatorname{cot}^{-1} x = \pi/2$$

$$\text{if } x > 0, y > 0, xy < 1$$

$$\text{if } x > 0, y > 0, xy > 1$$



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$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

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Straight line

Slope: If line makes an angle θ with +ve direction of x-axis then slope of line is $m = \tan\theta$.

2) Slope of line joining the points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y_2 - y_1}{x_2 - x_1}, x_2 - x_1 \neq 0$$

3) Slope of line $ax + by + c = 0$ is $-\frac{a}{b} = -\frac{\text{Coeff of } x}{\text{coeff of } y}$

Two lines are parallel if and only if their slopes are equal ($m_1 = m_2$)

Two lines are perpendicular if and only if product of their slope is -1. ($m_1 m_2 = -1$)

Equations of line:-

1) Slope point form: $y - y_1 = m(x - x_1)$

2) Two point form: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

3) intercept form: $y = mx + c$, where m is slope, c is y intercept of line.

4) Double intercept form: $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are x and y intercepts resp.

5) Normal form: $x \cos \alpha + y \sin \alpha = p$, Where p is length of perpendicular from origin.

- eqⁿ of x-axis is $y = 0$
- eqⁿ of y-axis is $x = 0$



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* Equation of lines parallel to x-axis is $y = \pm k$

* Equation of lines parallel to y-axis is $x = \pm k$

Plot



- Three points A,B,C are collinear $\Rightarrow$ slope of AB = slope of BC
= slope of AC.

- Distance formula :if A (x_1, y_1) , B (x_2, y_2) then

- Perpendicular distance of point p (x_1, y_1) to the line $ax+by+c=0$

$$\text{is } \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

- Distance between two parallel lines $ax+by+C_1=0$ & $ax+by+C_2=0$

$$\text{is } \left| \frac{C_1 - C_2}{\sqrt{a^2 + b^2}} \right|$$

- Acute angle θ between two lines having slope m_1 and m_2 is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$d(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Midpoint formula: - midpoint of line joining the points.

$$A(x_1, y_1) \text{ and } B(x_2, y_2) \text{ is } x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

- Centroid formula : $G(x_1, y_1, z_1)$ of ΔABC with vertices

$A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is

$$X = \frac{x_1 + x_2 + x_3}{3} \quad Y = \frac{y_1 + y_2 + y_3}{3}$$



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Statistics

Raw Data	Ungrouped Data	Grouped Data
a) Direct method $\bar{X} = \frac{\sum s_i}{N}$	$\bar{X} = \frac{\sum f_i s_i}{\sum f_i}$	$\bar{X} = \frac{\sum f_i s_i}{\sum f_i}$, where s_i is mid value of C.I
b) Step derivation method $\bar{X} = A + \frac{\sum s_i}{N}$, $d_i = x_i - A$ Where A is Assumed mean		Step deviation method $\bar{X} = A + \frac{\sum f_i d_i}{\sum f_i}$, $d_i = x_i - A$ Where A is Assumed mean.

Arithmetic Mean

Raw Data	Ungrouped Data	Grouped Data
a) Direct method $\bar{X} = \frac{\sum s_i}{N}$	$\bar{X} = \frac{\sum f_i s_i}{\sum f_i}$	$\bar{X} = \frac{\sum f_i s_i}{\sum f_i}$, where s_i is mid value of C.I



b)Step derivation method $\bar{x} = A + \frac{\sum S_i}{N}$ $d_i = x_i - A$ Where A is Assumed mean	Step deviation method $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$ Where A is Assumed mean.
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Median:

Raw Data	Grouped Data
Median $M = \left(\frac{N+1}{2}\right)^{th}$ Place observation, if N is odd Number.	$M = L + \left(\frac{\frac{N}{2} - f_c}{f_m}\right) \times C$ L= Lower limit of median class N = total frequency of medium class. fc = cumulative frequency of just previous of median class. C= class width.
$M = \frac{\left(\frac{N}{2}\right)^{th} o.b.c. + \left(\frac{N}{2} + 1\right)^{th} P.S.a.c. o.b.c.}{2};$ If N is even number	

Mean Deviation: → it is an absolute measure of dispersion.

It is the arithmetic mean of all the absolute deviations from any one of its average (means or median)



Mean Deviation (M.D)

Raw Data	Ungrouped Data	Grouped Data
i) M.D. about mean $= \frac{\sum d_i }{N}, d_i = x_i - \bar{X} $ $\bar{X}$ = mean of observations	i) M.D. about man: $= \frac{\sum f_i d_i }{N}$ $d_i = x_i - \bar{X} $	i) M.D. about mean $= \frac{\sum f_i d_i }{N}$ $d_i = x_i - \bar{X} $
ii) M.D. about median $= \frac{\sum d_i }{N}, d_i = x_i - M $ M= median of obs	ii) M.D. about median $= \frac{\sum f_i d_i }{N}$ $d_i = x_i - M $	ii) M.D. about median $= \frac{\sum f_i d_i }{N}$ $d_i = x_i - M $ x_i = mid values of C.I.

Standard Deviation (S.D) -S.D is most absolute measure of dispersion. It is denoted by 'σ'



Standard Deviation(S.D)

Raw data	Ungrouped data	Grouped data
$S.D (o) = \sqrt{\frac{\sum d_i^2}{N}}$ Where $d_i = (x_i - \bar{X})$ $\bar{X}$ = mean	$S.D = \sqrt{\frac{\sum f_i d_i^2}{N}}$ Where $d_i = (x_i - \bar{X})$ $N = \sum f_i$	$S.D = \sqrt{\frac{\sum f_i d_i^2}{N}}$ Where $d_i = (x_i - \bar{X})$ $N = \sum f_i$ X_i = mid values of C.I

$$\text{Variance} = (S.D)^2$$

$$\text{Co-efficient of S.D} = \frac{S.D}{\text{Mean}} = \frac{o}{\bar{s}}$$

$$\text{Co- efficient of Variance} = \frac{S.D}{\text{Mean}} \times 100 = \frac{o}{\bar{s}} \times 100$$

$$S.D (o) = \sqrt{\text{Variance}}$$



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Sem - II

Complex Number

i) **Defⁿ**: - A number of the form $a + ib$ where $a, b \in \mathbb{R}$ & $i = \sqrt{-1}$ is called a complex number, it is denoted by 'Z'.

Z = a + ib Where a is real part & b is imaginary part, i is imaginary number.

ii) **Conjugate of complex number:-**

if $Z = a + ib$ is a complex number, its conjugate is $\bar{Z} = a - ib$.

iii) **Modulus of complex number:-**

If $Z = a + ib$, its modulus $|z| = r = \sqrt{a^2 + b^2}$

iv) Co-ordinate (a,b) represents a complex number $a + ib$

v) **Argument of Complex number:-** If $Z = a + ib$, its Argument is

(Arg Z) or $\theta = \tan^{-1} (b/a)$

$\theta = \tan^{-1} |b/a| \rightarrow z$ lies in 1st quadrant, When $Z = a + ib$

$\theta = \pi - \tan^{-1} |b/a| \rightarrow z$ lies in 2nd quadrant, When $Z = -a + ib$

$\theta = \pi + \tan^{-1} |b/a| \rightarrow z$ lies in 3rd quadrant. When $Z = -a - ib$



$8 = 2n - \tan^{-1} |b/a| \rightarrow z$ lies in 4th quadrant. When $Z = a - ib$

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vi) **Polar form of complex number:-**

$$z = r (\cos \theta + i \sin \theta)$$

$$\text{Where } r = |z| = \sqrt{a^2 + b^2}$$

vii) **De-moivre's Theorem: →**

$$\text{i) } [\cos \theta + i \sin \theta]^n = \cos n\theta + i \sin n\theta$$

$$\text{ii) } [\cos \theta + i \sin \theta]^{-n} = \cos n\theta - i \sin n\theta$$

viii) **Exponential form:**

$$z = r e^{i\theta} = r [\cos \theta + i \sin \theta]$$

$$e^{i\theta} = [\cos \theta + i \sin \theta]$$

$$z = r e^{-i\theta} = r [\cos \theta - i \sin \theta]$$

$$e^{-i\theta} = [\cos \theta - i \sin \theta]$$

ix) **Euler's formula :-**

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

x) **Hyperbolic function:-**

$$\sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2}, \cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\sin i\theta = i \sinh \theta$$

$$\sinh i\theta = i \sin \theta$$

$$\cos i\theta = \cosh \theta$$

$$\cosh i\theta = \cos \theta$$

$$\tan i\theta = i \tanh \theta$$

$$\tanh i\theta = i \tan \theta$$



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Numerical methods

1) **Bisection Method:** if root lies in (a, b) then $x_1 = \frac{a+b}{2}$

2) **Regula falsi method:** if root lies in (a, b) then $x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}$

3) **Newton Raphson method:** if x_0 is initial root then $X_{n+1} = X_n - \left[\frac{f(x_n)}{f'(x_n)} \right]$

4) **Gauss elimination method:** To solve the equations

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$, eliminate the variables x,y,z by solving them simultaneously.

5) **Jacobis Method :** Consider the system of linear equations

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$ by rewriting the given equations $x = \frac{1}{a_1}[d_1 - b_1y - c_1z]$ ----- (1)

$$y = \frac{1}{b_2}[d_2 - a_2x - c_2z] \text{----- (2)}$$

$$z = \frac{1}{c_3}[d_3 - b_3y - a_3x] \text{----- (3)}$$

For first Iteration – put $x = y = z = 0$ in equation (1),(2)&(3). We get values of x_1, y_1 & z_1 .

For second Iteration – put $x = x_1, y = y_1, z = z_1$ in equation (1),(2)&(3). We get values of x_2, y_2 & z_2 , and so on.



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Gauss Seidel Method : : Consider the system of linear equations

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$ by rewriting the given equations x
 $= \frac{1}{a_1}[d_1 - b_1y - c_1z]$ ----- (1)

$$y = \frac{1}{b_2}[d_2 - a_2x - c_2z] \text{----- (2)}$$

$$z = \frac{1}{c_3}[d_3 - b_3y - a_3x] \text{----- (3)}$$

For first Iteration – put $y = z = 0$ in equation (1). We get values of x_1 .

Put $x = x_1$ & $z=0$ in equation (2). We get values of y_1 .

Put $x = x_1$ & $y= y_1$ in equation (3). We get values of z_1 .

For second Iteration — put $y = y_1$ & $z = z_1$ in equation (1). We get values of x_2 . Put $x = x_2$ & $z=z_1$ in equation (2). We get values of y_2 .

Put $x = x_2$ & $y= y_2$ in equation (3). We get values of z_2

And so on.

Function

Let A and B be two non-empty sets, then a relation

$f : A \rightarrow B$ is called a function, if every element of set A has unique images in set B.

Function is also called as map or mapping it is expressed as $y=f(x)$,

Where y is dependent variable, x is independent variable.

A set A is known as **Domain** of f and set B is known as the **co-domain** of f .

A set of image of elements of A is known as **Range** of f .



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Types of Functions:

- Constant Function : $f(x) = C$ where $c = \text{constant}$
- A function which consists only algebraic operations is called as Algebraic Function.
- Rational Function: $f(x) = \frac{P(x)}{Q(x)}$, $Q(x) \neq 0$ where $P(x), Q(x)$ are polynomials in X .
- Trigonometric Function: functions like $\sin x$, $\cos x$, $\tan x$...
- Implicit Function : $f(x, y) = 0$
- Parametric Function : $x = f(t)$, $y = g(t)$
- Composite function : $y = f[g(x)] \Rightarrow y = fog(x)$

Odd and Even function:

- a) If $f(-x) = -f(x)$, then $f(x)$ is odd function.
- b) If $f(-x) = f(x)$, then $f(x)$ is even function.

Inverse Function: if $f: A \rightarrow B$ is one-one, onto then a function $f: B \rightarrow A$ is called inverse function.

If $y = f(x) \Rightarrow x = f^{-1}(y)$.

Function which are not polynomial, instead they are logarithmic, trigonometric, exponential functions are called Transcended functions.

E.g. 1) $f(x) = \sin x + \cos x$ ----- Trigonometric function

2) $f(x) = \log x_e$ ----- Logarithmic function

3) $f(x) = e^x + 1$ ----- Exponential Function



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Limits

$$\lim_{x \rightarrow a} x = a$$

$$\lim_{x \rightarrow a} K = K$$

$$\lim_{x \rightarrow a} kx = ka, \\ \text{where } k \text{ is constant.}$$

$$\lim_{s \rightarrow a} \frac{s^n - a^n}{s - a} = na^{n-1}$$

$$\lim_{s \rightarrow a} \frac{s^n - a^n}{s^m - a^m} = \frac{n}{m} a^{n-m}$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1$$

$$\lim_{s \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$\lim_{s \rightarrow 0} \frac{e^x - 1}{x} = \log_e e = 1$$

$$\lim_{s \rightarrow 0} \frac{a^x - b^x}{x} = \log(a/b)$$

$$\lim_{s \rightarrow 0} \left(1 + \frac{x}{a}\right)^{a/x} = e$$

$$\lim_{s \rightarrow \infty} \left(1 + \frac{a}{s}\right)^{\frac{s}{a}} = e$$

$$\lim_{s \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{s \rightarrow \infty} \frac{1}{s} = 0$$

Derivative

□ first principle:-

$$\frac{dy}{ds} = f'(x) = \lim_{s \rightarrow 0} \frac{f(s+h) - f(s)}{h}$$

Standard Formulae:-

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$$\frac{d}{ds}(k) = 0 \quad k \in \mathbb{R}$$

$$\frac{d}{ds}(x) = 1$$

$$\frac{d}{ds}(x^n) = nx^{n-1}, n \in \mathbb{R}$$

$$\frac{d}{ds}\left(\frac{k}{s^n}\right) = \frac{-kn}{s^{n+1}}, k \in \mathbb{R}, K \neq 0$$

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$$\frac{d}{ds}(a^s) = a^s \log a \quad a > 0$$

$$\frac{d}{ds}(\log x) = \frac{1}{s}$$

$$\frac{d}{ds}(\sqrt{x}) = \frac{1}{2\sqrt{s}}$$

$$\frac{d}{ds}(\sin x) = \cos x$$

$$\frac{d}{ds}(\tan x) = \sec^2 x$$

$$\frac{d}{ds}(\sec x) = \sec x \cdot \tan x$$

$$\frac{d}{ds}(\sin^{-1} x) = \frac{1}{\sqrt{1-s^2}}$$

$$\frac{d}{ds}(\tan^{-1} x) = \frac{1}{1+s^2}$$

$$\frac{d}{ds}(\sec^{-1} x) = \frac{1}{s \cdot \sqrt{s^2-1}}$$

$$\frac{d}{ds}(e^s) = e^s$$

$$\frac{d}{ds} \log_a x = \frac{1}{s \log a}$$

$$\frac{d}{ds} \left(\frac{1}{x} \right) = \frac{-1}{s^2}$$

$$\frac{d}{ds}(\cos x) = -\sin x$$

$$\frac{d}{ds}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{ds}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$$

$$\frac{d}{ds}(\cos^{-1} x) = \frac{-1}{\sqrt{1-s^2}}$$

$$\frac{d}{ds}(\cot^{-1} x) = \frac{-1}{1+s^2}$$

$$\frac{d}{ds}(\operatorname{cosec}^{-1} x) = \frac{-1}{s \cdot \sqrt{s^2-1}}$$

□ Derivative of sum & difference of function:-

If U & V are two differentiable function of x. If $y = u \pm v$ then

$$\frac{dy}{ds} = \frac{d}{ds}(u \pm v) = \frac{d}{ds}(u) \pm \frac{d}{ds}(v)$$

□ Derivative of the product function :-

If u & v are two differentiable function of x, If $y = uv$

$$\text{Then } \frac{dy}{ds} = \frac{d(uv)}{ds} = u \frac{dv}{ds} + v \frac{du}{ds}$$



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□ **Derivative of the quotient :**

If u & v are different function of x, If $y = \frac{u}{v}$ then

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

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□ **Chain Rule(composite function) :-**

If y is a function of u & u is function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

□ **Derivative of Parametric function :-**

STO



APPLIED MATHEMATICS (SEM-III)

Application of Derivative

$\frac{dy}{dx}$ Represents a **slope of tangent** to the curve $y = f(x)$ at the point $p(x_1, y_1)$ on it.

equation of tangent to the curve $y = f(x)$ at $p(x_1, y_1)$ is

$$y - y_1 = m(x - x_1), \quad m = \frac{dy}{dx}$$

equation of normal to the curve is : $y - y_1 = -\frac{1}{m}(x - x_1), \quad m = \frac{dy}{dx}$

Condition of Maxima and minima :

a) A function $y = f(x)$ is **maximum** at $x = a$ if 1) $f'(x) = 0$

$$2) f''(a) < 0$$

b) A function $y = f(x)$ is **minimum** at $x = a$ if 1) $f'(x) = 0$

$$\text{Radius of curvature : } \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{|\frac{d^2y}{dx^2}|}$$

Probability

For any event A, $P(A) = \frac{n(A)}{n(S)} \quad 0 \leq P(A) \leq 1$

$$P(A^c) = 1 - P(A)$$

$$P(A^c \cup B^c) = 1 - P(A \cap B)$$

$$P(A^c \cup B^c) = 1 - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive

Plot



$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(B) > 0$$

$P(A \cap B) = P(A) \cdot P(B)$ if A & B are independent event.

- **Binomial distribution :** $p(x = r) = {}^n C_r p^r q^{n-r}$

Where P = Probability of success in single trials. $q = 1 - P$

n = no. of trials

r = no. of success in n -trials

- **Poisson's distribution :**

$$p(x = r) = \frac{e^{-m} m^r}{r!}$$

Mean : $m = np$

Variance (V) = mean = np

S.D = $\sigma = \sqrt{\text{variance}}$

- **Normal distribution**

If z is standard normal variable then, $Z = \frac{x - \mu}{\sigma}$

Where x is binomial variable

μ = mean = np

σ = standard deviation



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Integration

Definition: - If $\frac{d}{ds} [f(x)] = g(x)$ then $\int g(x)dx = f(x) + c$

f(x)	Integration	f(x)	Integration
1	x	cot x	log sinx
k	kx	secx	log secx + tanx
x ⁿ	$\frac{s^n+1}{n+1}$	cosecx	log cosecx - cotx
$\frac{1}{x}$	log x	$\frac{1}{1+x^2}$	tan ⁻¹ x
e ^s	e ^s	$\frac{1}{\sqrt{1-x^2}}$	sin ⁻¹ (x)
a ^s	$\frac{a^x}{\log a}$	$\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
sin x	-cosx	$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \log \left \frac{x-a}{x+a} \right $
cos x	sin x	$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \log \left \frac{a+x}{a-x} \right $
sec ² x	tan x	$\frac{1}{\sqrt{x^2 + a^2}}$	log x + $\sqrt{x^2 + a^2}$
cosec ² x	-cot x	$\frac{1}{\sqrt{x^2 - a^2}}$	log x + $\sqrt{x^2 - a^2}$
secx.tanx	secx	$\frac{1}{\sqrt{a^2 - x^2}}$	sin ⁻¹ ($\frac{x}{a}$)
cosecx. cotx	-cosecx	$\frac{1}{s^2 - a^2}$	$\frac{1}{2a} \log \left \frac{s-a}{s+a} \right $
tan x	log secx	$\frac{1}{\sqrt{a^2 + s^2}}$	log x + $\sqrt{a^2 + x^2}$
$\sqrt{x^2 + a^2}$	$\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log x + \sqrt{x^2 + a^2} $		
$\sqrt{x^2 - a^2}$	$\frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log x + \sqrt{x^2 - a^2} $		

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$\sqrt{a^2 - x^2}$	$\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)$
--------------------	--

Formulea:

STO



$$\int \frac{f^{F(x)}}{f(s)} dx = \log |f(x)| + c$$

$$\int \frac{f^{F(x)}}{f(s)} dx = 2 \int f(x) + c$$

Integration by parts: $\int u \cdot v dx = u \int v dx - \int \left[\frac{du}{dx} \cdot \int v dx \right] dx$.

❖ Substitution to remove the square root in the integration :-

If integral contains

substitute

1) $\sqrt{a^2 - x^2}$

$x = a \sin \theta$ or $x = a \cos \theta$

2) $\sqrt{x^2 - a^2}$

$x = a \sec \theta$ or $x = a \csc \theta$

3) $\sqrt{a^2 + x^2}$

$x = a \tan \theta$ or $x = a \cot \theta$

5) $\int \frac{(a-s)}{s}$

$x = a \sin^2 \theta$ or $x =$

6) $\sqrt{2ax - x^2}$

$2a \sin^2 \theta$ or $x = a \cos$

7) $\int \frac{a-s}{a+s}$

θ

8) $\int \frac{a^2 - s^2}{a^2 + s^2}$

9) $\sqrt{2ax - x^2}$

Definite Integration

If $\int_a^b f(x) dx = g(x) + c$ then $\int_a^b f(x) dx = [g(x)]_a^b = g(b) - g(a)$

Properties of Indefinite Integration -

1) $\int_a^b f(x) dx = \int_a^b f(t) dt$

2) $\int_a^b f(x) dx = - \int_b^a f(x) dx$

3) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad \dots a < c < b$

4) $\int_a^b f(x) dx =$



$$\int_0^b f(a + b - x) dx$$

$$5) \int_0^a f(x) dx = \int_0^a f(a - x) dx$$

$$6) \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(2a - x) dx$$

SL01



$$7) \int_{-a}^a f(x) dx = 0 \quad \rightarrow \text{if } f(x) \text{ is odd}$$

$$= 2 \int_0^a f(x) dx \rightarrow \text{if } f(x) \text{ is even}$$

Applications of definite integration -

Area bounded by the curve $y = f(x)$, x -axis and ordinate

$x = a$ and $x = b$ is, $A = \int_a^b y dx$

Area between two curves $y = f(x)$ and $y_1 = g(x)$ is given by

$$A = \int_a^b (y - y_1) dx$$

Differential equation (D.E.)

- ❖ An equation involving differential co-efficient is called D.E.

e.g. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right) - y = 0$

Order: order of highest derivative in given D.E.

Degree: degree of highest order derivative of D.E.

Note: order of D.E. is always positive integer

To find order and degree make D.E. Free from radicals and Fractions.

- ❖ Order of D.E. is equal to the number of arbitrary constants.
- ❖ Formation of D.E -
 - 1) Find out number of arbitrary constants of eqn.
 - 2) Try to eliminate constants by differentiating eqn.
- ❖ A solution in which number of arbitrary constants is exactly equal to the order of the D.E. is called General solution.



❖ **Variable separable form :-**

In this method write given diff eqn in the form

$$f(x) dx = f(y) dy \quad [\text{i.e. separate variables}]$$

Then by integrating we get sol^n .

❖ **Homogeneous D.E** -. A differential equation in which degree of each term is same is called homogeneous D.E.

SLO



❖ Its solution is obtained by putting $y = vx$
Diff. w.r.t.x.
 $\therefore dy = v + x \frac{dv}{dx}$

❖ **Exact Diff Eqⁿ**
An eqⁿ of the form $M dx + N dy = 0$ is called exact D.E.

If
where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ is partial derivative of M w.r.to.y keeping x constant
its solⁿ is given by
taking x y constant
is partial derivative of N w.r.to.x keeping y constant

$\int M dx + \int N dy = c$
It is the D.E. of the form $P \frac{dy}{dx} + Qy = R$ (i.e. terms of y only)

❖ **Linear Diff eqn**

Where P and Q are constants or functions of x and constant .
Its solution is obtained by

$y (I.F.) = \int Q \cdot (I.F.) dx + c$
Where I.F. = $e^{\int P dx}$

$e^{\int P dx} f(s) = f(x)$
It is the eqⁿ of the form $\frac{dy}{dx} + py = Q(y)^n$

❖ **Bernoulli's Diff. eqⁿ :-**

Where P and Q are constants or functions of x only



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BASIC PHYSICS

(FOR ALL BRANCHES)

ELASTICITY

S.I.UNITS AND FORMULAE (SEM I)

Topic 1: Properties of Solids

Stress	N/m^2	F/A
Strain	Unit less	Change in dimension/original dimension
Young's Modulus (Y)	N/m^2	$Y = FL/AI$, $Y = mg/\Delta L$
Bulk Modulus (K)	N/m^2	$K = V.(dP/dV)$
Compressibility (1/K)	$1/\text{N/m}^2$	$1/K = 1/V.(dV/dP)$
Modulus of Rigidity (η)	N/m^2	$\eta = F/A\theta$
Relation of (Y ,K , η)	-----	$Y = 9K\eta / 3K + \eta$
Poisson's Ratio (δ)	Unit less	$-\Delta D.L/D.L$



Topic 2: Properties of Liquids

Pressure (P)	N/m^2	F/A
Velocity gradient (V)	/sec (per second)	dv/dx
Force (F)	newton (N)	Stokes Formula $F = 6 \pi \eta r v$
Area (A)	m^2	
Newton's Law of Viscosity OR Viscous Force	newton (N)	$F = \eta A.(dv/dx)$
Coefficient of Viscosity (η)	N Sec/m^2	$\eta = 2/9 r^2 g(d-p)/v$
Critical Velocity (V_c)	m/s	$V_c = R.\eta/p.r$
	Kg/m^3	
	N/m	$T = rhpg/2\cos\theta$
Acceleration due to gravity (g)	m/s^2	
Angle of contact (θ)	Degree ($^\circ$)	
Height (h)	m	

Temperature gradient	$^\circ \text{C/m}$	$\theta_1 - \theta_2 / d$
Coefficient of thermal conductivity (K)	$\text{Kcal/m}^\circ \text{C Sec}$ OR $\text{Watt/m}^\circ \text{C}$	$K = Q d / A.(\theta_1 - \theta_2).t$
Temperature (T)	$^\circ \text{C}$	
Pressure (P)	atm.	
Volume (V)	m^3	
Joule's Constant (J)	J/kcal	
Specific heat of gas at constant Pressure (C_p)	$\text{J/kg}^\circ \text{C}$	
Specific heat of gas at constant Volume (C_v)	$\text{J/kg}^\circ \text{C}$	
Adiabatic Index (γ)	-----	$C_p/C_v > 1$
Relation between (C, F, T)	$^\circ \text{C}, ^\circ \text{F}, ^\circ \text{K}$	$C = 5/9(F-32), F = (9/5 \cdot C) + 32, T = C + 273$
Boyle's Law	-----	$P_1 V_1 = P_2 V_2$
Charles's Law	-----	$V_1/T_1 = V_2/T_2$
Gay Lussac's Law	-----	$P_1/T_1 = P_2/T_2$
Perfect Gas Equation	-----	$PV = RT$
Mayer's Relation	-----	$C_p - C_v = R/J$



Topic 4: Optics

Velocity of light in air or glass (V_a & V_g)	m/s	${}_a\mu_g = V_a/V_g$
Velocity of light in air or water (V_a & V_w)	m/s	${}_a\mu_w = V_a/V_w$
Time (t)	sec	
Critical Angle (θ_c)	Degree min.sec. ($<'''$)	$\theta_c = \sin^{-1}C$
Numerical Aperture(N.A)	Unit less	$NA = \sqrt{\mu_{core}^2 - \mu_{clad}^2}$
Acceptance Angle (θ_m)	Degree ($<$)	$\theta_m = \sin^{-1}(N.A)$
Refractive Index (μ)	Unit less	Snell's Law (μ)= $\sin i/\sin r$
Prism Formula (μ)	-----	$\mu = \sin(A + \delta m/2) / \sin(A/2)$

Amplitude (a)	meter (m)	
Wavelength (λ)	meter (m)	
Frequency (n)	Hertz (Hz)	$n=1/T$
Periodic Time (T)	Sec.(s)	$T=2\pi/\omega$
Length of air column (l)	meter (m)	
Corrected Length (L)	meter (m)	
Diameter (d)	meter (m)	
Velocity of wave (V)	m/s	$V=n\lambda$
Angular Velocity (ω)	rad/sec	$\omega=2\pi n$
Displacement of S.H.M (y) mean position, general eq ⁿ , extreme position ($\alpha=90^\circ$)	meter (m)	$y=asin\omega t, y=asin(\omega t+\alpha), y=acos\omega t$ where α is epoch=0
Velocity of S.H.M (v)	m/s	$V=\omega\sqrt{a^2-y^2}$
Acceleration of S.H.M (a)	m/s ²	$a=\omega^2 y$
Resonance Formula		$V=4nL, V=4n(l+0.3D)$



SLOT



SEM – II

APPLIED PHYSICS

(ME/CE/CO)



S .I.UNITS AND FORMULAE (SEM II) MECHANICAL

Topic 1.1: Rectilinear Motion & Angular Motion

Distance	meter	
Velocity	meter/second	$V=S/t$
Acceleration	m/s^2	$a=v-u/t$
Time	Second	
Displacement	Meter	$S=V*t$
Kinematical Equation		$V=u+at, S=ut+1/2at^2, v^2=u^2+2as$
Motion Under Gravity		$V=u+gt, S=ut+1/2gt^2, v^2=u^2+2gs$
Motion Under Gravity		$V=u-gt, S=ut-1/2gt^2, v^2=u^2-2gs$
		$S^{nth} = a/2(2n-1)$
Angular Displacement	radian	
Angular Velocity	radian/second	
Angular Acceleration	radian/second ²	
Angular Equation		$\omega_f = \omega_i + \alpha t, \theta = \omega_i t + 1/2 \alpha t^2, \omega^2 = \omega_i^2 + 2\alpha\theta$
		$\theta^{nth} = \alpha/2(2n-1)$

Momentum	Kg m/s OR Ns	Mass*Velocity
Impulse	Kg m/s OR Ns	$m(v-u)$
Impulsive Force	Kg m OR N	$m(v-u)/t$
Recoil of gun		Recoil of gun = $m_1 v_1 / m_2 v_2$
Work	Nm	Work = $F*S$
Power	Watt	$F*S/t$
Energy	Joule	
Kinetic Energy	Joule	$K.E = 1/2 mv^2$
Potential Energy	Joule	$P.E = mgh$

Time of flight	Second	$T = 2u \sin \alpha / g$
Horizontal Range	Meter	$R = u^2 \sin 2\alpha / g$
Maximum Height	Meter	$H = u^2 \sin^2 \alpha / g$
Centripetal Acceleration	m/s^2	$a_c = v^2 / r$
Centripetal Force	N	$F_c = mv^2 / r$

Joule's Law	Joule	$H = I^2 t R$
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Energy of Photon	Joule	$E = h\nu$
Velocity of Light	m/s	$C = \lambda \nu$
Threshold Wavelength	Meter	$\lambda_0 = hc / w_0$
Maximum Kinetic Energy	Joule	$K.E_{Max} = h(\nu - \nu_0)$
Work Function	Joule second	$w_0 = h \nu_0$

Topic 4.2: X- Rays

Minimum Wavelength	Meter	$\lambda_{min} = 12400/V$
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SLOT



S .I.UNITS AND FORMULAE (SEM II) CIVL

Topic 1.1: Rectilinear Motion & Angular Motion

Distance	meter	
Velocity	meter/second	$V=S/t$
Acceleration	m/s^2	$a=v-u/t$
Time	Second	
Displacement	Meter	$S=V*t$
Kinematical Equation		$V=u+at, S=ut+1/2at^2, v^2=u^2+2as$
Motion Under Gravity		$V=u+gt, S=ut+1/2gt^2, v^2=u^2+2gs$
Motion Under Gravity		$V=u-gt, S=ut-1/2gt^2, v^2=u^2-2gs$
		$S^{nth} = a/2(2n-1)$
Angular Displacement	radian	
Angular Velocity	radian/second	
Angular Acceleration	radian/second ²	
Angular Equation		$\omega_f = \omega_i + \alpha t, \theta = \omega_i t + 1/2 \alpha t^2, \omega^2 = \omega_i^2 + 2\alpha\theta$
		$\theta^{nth} = \alpha/2(2n-1)$

Momentum	Kg m/s OR Ns	Mass*Velocity
Impulse	Kg m/s OR Ns	$m(v-u)$
Impulsive Force	Kg m/s OR N	$m(v-u)/t$
Recoil of gun		Recoil of gun = $m_1 v_1 / m_2 v_2$
Work	Nm	Work = $F*S$
Power	Watt	$F*S/t$
Energy	Joule	
Kinetic Energy	Joule	$K.E = 1/2 mv^2$
Potential Energy	Joule	$P.E = mgh$

Time of flight	second	$T=2u\sin\alpha/g$
Horizontal Range	Meter	$R=u^2\sin2\alpha/g$
Maximum Height	Meter	$H= u^2 \sin^2\alpha/g$
Centripetal Acceleration	m/s^2	$a_c=v^2/r$
Centripetal Force	N	$F_c= mv^2/r$

Reverberation time	second	$t = 0.164V/\Sigma aS$
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Topic 4.1: Photo electricity

Energy of Photon	Joule	$E = h\nu$
Velocity of Light	m/s	$C = c/\lambda$
Threshold Wavelength	Meter	$\lambda_0 = hc / w_0$
Maximum Kinetic Energy	Joule	$K.E_{Max} = h(\nu - \nu_0)$
Work Function	Joule second	$w_0 = h\nu_0$



SLOT



Topic 4.2: X- Rays

Minimum Wavelength	Meter	$\lambda_{\min} = 12400/V$
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S .I.UNITS AND FORMULAE (SEM II) EJ/IF/CO

1. Basic Electric circuit.

Resistance	Ohm	$R = \frac{L}{A}$
Resistivity	Ohm meter	$\rho = R \cdot a / L$
Conductivity	$1/\Omega m$	$\sigma = 1/\rho$
Ohms law		$V = IR$
Capacitance	Farad	$C = Q/V$
		$C = KA\epsilon_0/d$

Energy of Photon	Joule	$E = h\nu$
Velocity of Light	m/s	$C = \lambda \nu$
Threshold Wavelength	Meter	$\lambda_0 = hc / w_0$
Maximum Kinetic Energy	Joule	$K.E_{\max} = h(\nu - \nu_0)$
Work Function	Joule second	$w_0 = h\nu_0$

Minimum Wavelength	Meter	$\lambda_{\min} = 12400/V$
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SLOT



SLOT



SEM – I BASIC CHEMISTRY

(ALL BRANCHES)

50 MARKS

IMPORTANT FORMULAE

(FOR ALL BRANCHES)

TOPIC-1 CHEMICAL BONDING

1. Atomic Number = No. of Protons = No. of Electrons.

$$Z = p = e^{-}$$

2. Atomic Mass Number = No. of Protons + No. of Neutrons

$$A = p + N$$

3. No. of Neutrons,

$$N = A - Z$$

TOPIC-2 ELECTROCHEMISTRY

1. $W = ZQ$ or $W = ZCt$

2. Wt. of substance A deposited = C.E (eq. wt) of substance A



Wt. of substance B deposited C.E (eq. wt) of substance B

3. $C.E \text{ (eq.wt)} = Z \text{ (ECE)} \times 96500$

4. $Eq. \text{ wt} = \frac{\text{Atomic weight}}{\text{Valency}}$

Valency

5. $Eq. \text{ wt of base} = \frac{\text{Molecular wt. of base}}{\text{Acidity of Base}}$

Acidity of Base

6. $Eq. \text{ wt of acid} = \frac{\text{Molecular wt. of acid}}{\text{Basicity of acid}}$

Basicity of acid

7. $pH = -\log_{10}[H^+]$

8. $pOH = -\log_{10}[OH^-]$

9. $k_w = [H^+][OH^-] = 1 \times 10^{-14}$



WINTER – 2018 EXAMINATION

Subject Name: English

Model Answer

Subject Code: 22101

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions, credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	(a)	Attempt any five of the following: State the views of JRD Tata about a leader. Ans. According to JRD Tata, leadership meant motivating others. So leader should motivate others. To be a leader, one has to get to lead human beings with affection. The leader should identify talent easily. And once he/she is confident that a manager would perform, he/she should give the manager a long hope.	(10) 02
	(b)	Describe the role of Mr. Lal as a doer in the organization. Ans. When Lal started the organization, he was a 'doer' in the organization. He used to 'do' everything including a coding, system architecture, accounting, administration, back office or business development. He was solely responsible for these activities. The 'doer' phase helped him to sustain, as he had to curtail costs. It also helped him to know all the nuances of the business. As he said without Doer stage, he would've never understood higher management thoughts.	02
	(c)	Describe the family condition of Shiva Kumar during his childhood and school life. Ans. The family condition of Shiva Kumar during his childhood and school life was not financially well off. His father was an out-of-work lorry driver who did not have Rs.15,000 to pay Shiva's annual fees. Shiva's mother sold her jewellery for his admission in Maruthi Vidyalaya, an ICSE School. Shiva and his elder sister Yogeshwari used to sell the garlands at traffic signals and other places after their school. Shiva used to sell the newspapers. Despite all this hard work, money was insufficient for the family.	02



Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	(d)	Define the term: Electronic Waste. Ans. Electronic waste or e-waste is the discarded electronic appliances like computers, mobile phones, television sets, refrigerators etc.	02
	(e)	Name the academy started by Arunima Sinha and its motto. Ans. Arunima Sinha started Shahid Chandrashekhkar Azad Viklang Khel Academy (Freedom Fighter Chandrashekhkar Azad Sports Academy for Disabled Children). The motto of this institute is to provide the best sports facilities to handicapped children, especially the poor ones.	02
	(f)	Describe the preparation stage of Cristiano Ronaldo. Ans. Cristiano Ronaldo used to practice daily from morning to night. Ronaldo had to go away from his family for better training. He was depressed and home sick. He used to call home and cry daily. He wanted to quit but his determination was stronger than his self-doubt. He focused all his energy in improving his game. Cristiano Ronaldo got selected for Portugal's most famous club "Sporting Lisbon".	02
	(g)	Explain the importance of wearing helmet while riding a two-wheeler. Ans. Wearing helmet is one of the safety measures one should follow while riding a bike or two-wheeler. Many road accidents cause serious injuries like head or brain damage or deaths. It is very necessary to wear helmet to prevent such physical harm as it gives protection. It reduces the chances of serious brain injury as it protects our head from hard surface during the accident. It also protects our eyes from dust or terrible light, forehead and face if one falls down. Thus, helmet is important to save our life as 'Prevention is better than cure'.	



2. **Attempt any three of the following:** (12)

(a) **Summarize the suggestions given by the author to make one's life remarkable.**

Ans: To make one's life remarkable, the author suggests reading great books that make us believe we can be 'remarkable'. We should nurture the company of people whose presence, comments and life style makes us believe that we can be 'remarkable'. Also we need to meditate, exercise and eat nourishing food that provides us vital energy to live a remarkable life. Whatever happens, we must believe in ourselves.

04

b) **Describe 'the memory of that dreadful night' in the mind of Anish's father.**

Ans: When Anish's father came to know about Anish's accident, he quickly went to the accident spot. After the accident, Anish was lying unconscious on the road and the bike was on the other side of the



Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		road. His father took him to the nearby hospital. There was urgency in the voice of the nurse sent by the doctor to call him. He felt a surge of anxiety as he entered the doctor's cabin. The doctor told his father that Anish's brain had been severely injured and he would collapse in a couple of hours. However, he was not ready to accept that. So he shifted Anish immediately to a leading hospital. The neurosurgeon there, examined him and decided to go for a surgery. Alone he was waiting outside the operation theatre at 2.30 am, weeping continuously. Wild thoughts like "In what state is he going to be after the surgery? Is he going to be alive? Is he going to be normal? Will he ever speak again? Is he going to walk, or will he have to use the wheelchair?" ran in his mind. After the surgery, the doctor told him that in a scale of 1 to 15, where a normal person would be rated 15, Anish could be rated only three. He was then shifted to a special room with 24-hour special nurses to take care of him. Tubes were inserted into his body to aid breathing, feeding and urination. The right side of his skull was cut open and kept in his abdomen for future use. The only movements in his body were that of his head, which moved right and left and his eye balls. He was thinking why that plight had befallen his son.	04
	(c)	Summarize the contribution of JRD Tata as a philanthropist. Ans: JRD Tata was a philanthropist who wanted India to be a happy country and did all he could to make it so. He was also a man of great sensitivity and was pained by the poverty he saw around him and wanted strongly to alleviate it. He was a patron of the sciences and the arts. As a patron of the arts, JRD was revered by India's artists, sculptors and performing artist. Under JRD's tutelage, the Tatas became the biggest buyers, promoters and supporters of the art world in India. As a philanthropist, JRD Tata built up the tremendously active Tata Charitable Trust and kept it alive. Entrepreneurs like Sir Homi Mody, Sir Ardeshir Dalal, Sir Jehangir Ghandy, Russi Mody, Sumant Moolgaokar and Darbari Seth, etc. were developed under JRD Tata's supportive climate. Scientists of international repute such as Homi Bhabha, leading lawyers such as J. D. Choksi and Nani Palkhivala and economists such as John Matthai, A. D. Shroff, D. R. Pendse and Freddie Mehta could flourish under JRD Tata's guidance.	04
	(d)	Justify the statement 'He (Gurmeet Singh) is like God'. Ans: Gurmeet Singh is a true Samaritan. Someone among the abandoned patients says, 'He is like God'. This statement proves itself when we learn about Gurmeet Singh's contribution as a philanthropist. Gurmeet Singh, once visited the Government hospital ward with poor lady and her scalded son, he was moved by the scene there. At that moment, he decided to do something for the patients of the ward. He then started providing these abandoned patients with food and medicines. He also	

Q. Sub

Answer

Marking

72| Contact us:

9372844023/7738376831

Plot No. 46, sector 5

Near MSEB sub station, Kharghar



3.

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Sam : I would also like to read such books. You know even I am fond of reading. Sunil : Yes, you can take my copy as I finish reading. Along with fiction, reading non-fiction is also beneficial to gain knowledge and information. Sam : Yes, you are right. Reading magazines that are published for specific and general purposes helps us a lot in our professional and social life. Sunil : Right! Even reading newspaper daily keeps us updated about what is happening around. Sam : True! Inspirational and motivational reading also helps us keeping our emotional balance. Sunil : Very true! Reading itself is a huge source of acquiring knowledge. Sam : Exactly! Reading is important not only for students but for each and every person to understand the life in its true sense. Well, I will take your leave now. Bye! Sunil : So keep reading. Bye! (Note: Contents may vary.)	04
	(c)	Write a paragraph on 'A Visit to a Historical Place'. (75 words) Ans: What a historical visit it was! Last Monday, I visited Shaniwarwada which is located at Bajirao Road in Pune. I saw beautiful walls. I was astonished when I saw well designed Delhi Darwaja which is the main gate of Shaniwarwada. I entered Shaniwarwada with the curiosity of what I will get to observe inside. I came to know that Shaniwarwada was originally the seven-storied capital building of the Peshwas of the Maratha Empire. Then I saw beautiful Mahal where Peshwas used to conduct meetings. I also saw Khidki Darwaja, Mastani Darwaja, Narayan Darwaja and Ganesh Darwaja. Really it was a nice experience. (Note: Contents may vary.)	04
	(d)	Write a paragraph on 'Technical specifications of a Mobile Phone'. (75 words) Ans: Mobile phone is an electronic device which is used for telecommunication. It is also used for entertainment like listening to songs, watching videos, internet and other related activities. A mobile phone is made up of different components like CPU, battery, an input mechanism like keypad for feature phones and touch screens for most smartphones, display, speakers, SIM cards and R-UIM cards. A CPU is the processor	



of phones. A battery provides the power source for the phone functions.
An input mechanism, like keypad for feature phones and touch screens for most smartphones, allows the user to interact with the phone.
A display shows the user's typing, text messages, contacts, and more. **04 (Note: Contents may vary.)**

STOFT



Q. No.	Sub Q. N.	Answer	Marking Scheme
4.		Attempt any three of the following:	(12)
	(a)	Make sentences using the following collocation:	
	(i)	Pay attention Ans: Most of the electronic companies <u>pay attention</u> to new innovations.	01
	(ii)	Come to an end Ans: The debate <u>came to an end</u> after half an hour.	01
	(iii)	Make progress Ans: He wants to <u>make progress</u> in his business by hook or by crook.	01
	(iv)	Have sympathy Ans: The rich should <u>have sympathy</u> for the poor.	01
	(b)	Replace the underlined words with correct words and rewrite the sentences :	
	(i)	There are various fruits kept in the basket like <u>banana</u> , <u>apple</u> and <u>pair</u> . Ans: There are various fruits kept in the basket like banana, apple and <u>pear</u> .	01
	(ii)	Have you read the story of the tortoise and the <u>hair</u> ? Ans: Have you read the story of the tortoise and the <u>hare</u> ?	01
	(iii)	The glass <u>brakes</u> easily. Ans: The glass <u>breaks</u> easily.	01
	(iv)	You are not <u>aloud</u> to play here. Ans: You are not <u>allowed</u> to play here.	01
	(c)	Write the meanings of following idioms and make sentences of your own:	
	(i)	Burn the midnight oil Ans: Meaning: To work late into the night Sentence: Most of the students <u>burn the midnight oil</u> during the examination period.	01 01
	(ii)	Castles in the air Ans: Meaning: Imaginary schemes Sentence: Swara builds <u>castles in the air</u> and actually does nothing to accomplish it.	01 01



Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	(d)	Choose the correctly spelt word from the following:	
	(i)	courageus, corageous, courageous Ans: courageous	01
	(ii)	ninth, nineth, nintth Ans: ninth/ nineth	01
	(iii)	beautyful, beautiful, beautifull Ans: beautiful	01
	(iv)	leafs, leaffs, leaves. Ans: leaves	01
	(e)	Change the voice:	
	(i)	Let this post be advertised. Ans: Advertise this post.	01
	(ii)	The guard refused the bribe. Ans: The bribe was refused by the guard.	01
	(iii)	Who ate the last cookie? Ans: By whom was the last cookie eaten?	01
	(iv)	A road show was being performed by them. Ans: They were performing a road show.	01
5.	(A)	Attempt <u>any two</u> of the following:	(08)
	(i)	Prepare a vote of thanks for the Annual Sports Day held in your institute. Ans: Good afternoon ladies and gentlemen! I am standing before you to propose vote of thanks. At the outset, I would like to thank our honorable Founder President, for giving us an opportunity to organize Annual Sports Day. I am also thankful to our principal who motivated and supported us for the success of this event. I thank the referees from the bottom of my heart for taking out some time for us and gracing the event by their presence. I express my heartfelt thanks to all the teachers who inspired us during this Annual Sports Day. I thank all the organisers, coordinators, volunteers and my colleagues; without their support the event would not have been successful. Before I conclude, I extend my thanks to all the participants for being a part of this Annual Sports Day. Thank you all. (Note: Contents may vary.)	04



SLOT



Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	(ii)	Draft a speech for farewell function of Mr. Pawar, Manager of Production Department of Apex Company, Mumbai, who is retiring this month end. Ans: Good afternoon ladies and gentleman! Today we have gathered here for bidding farewell to Mr. Pawar, Manager of Production Department of our Apex Company, Mumbai. He has put in total 19 years of service as a worker and 05 years as a manager in the Production department of our company. He has worked with enthusiasm and devotion and contributed positively for the development of the Production department and our company as well. He shared a good rapport with workers. He will retire this month end after putting 24 years of dedicated service. I wish him good health, peace of mind and lots of happiness in the years to come. Once again I take this opportunity to wish Mr. Pawar a happy retired life. May god bless you with good health and a long life. Thank you. 04 (Note: Contents may vary.)	
	(iii)	Draft a speech for introducing the guest speaker, Dr. Sinha, an expert on presentation skills for a seminar organised for the third year students in your institute. Ans: Good morning ladies and gentlemen! Welcome to the seminar on 'Presentation Skills'. I take this opportunity to welcome today's Chief Guest Dr. Sinha, an expert in presentation skills for this seminar today. He has completed M. A. in English from Pune University. Later on he pursued his MBA degree from IIM Lukhnow. It is an honour and a privilege for us Sir to have you with us today. I also welcome the dignitaries on and off the dais, teachers and my dear friends. Today we have gathered here for the seminar arranged for the third year students on 'Presentation Skills'. I am sure we will be enriched by the seminar. Once again I welcome all of you for this function. (Note: Contents may vary.)	04
5	(B)	Change into indirect speech (any two):	(04)
	(i)	She said, "How nice of you!" Ans: She exclaimed that I/he/she/we/they was/were very nice. OR She exclaimed that it was very nice of me/him/her/us/them.	02
	(ii)	The teacher said to the student, "Have you purchased a computer?" Ans: The teacher asked the student if/whether he/she had purchased a computer.	02
	(iii)	Neeta said to me, "The Sun rises in the East." Ans: Neeta told me that the Sun rises in the East.	02



Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	(a)	Attempt <u>any two</u> of the following:	(12)
	(i)	Do as directed: An vice chancellor has agreed to promote him to the post of an reader. (Rewrite the sentence using the correct articles in place of the underlined articles.) Ans: <u>The</u> vice chancellor has agreed to promote him to the post of <u>a</u> reader.	02
	(ii)	Ashok believes that he will get success <u>when</u> he works hard. He will definitely get the job <u>unless</u> he deserves it. (Rewrite the sentence using the correct conjunctions in place of the underlined conjunctions.) Ans: Ashok believes that he will get success <u>if</u> he works hard. He will definitely get the job <u>if/because</u> he deserves it.	02
	(iii)	The new trainees are generally seen practising on <u>the</u> river bank. They are not allowed to go <u>at a</u> certain point. (Rewrite the sentence using the correct prepositions in place of the underlined prepositions.) Ans: The new trainees are generally seen practising <u>at</u> the river bank. They are not allowed to go <u>beyond</u> a certain point.	02
	(B)	Read the following passage and answer the given questions: We left Dehradun early in the morning and stopped by for breakfast at Mussouri. From Mussouri the picturesque road heads north to Yamuna bridge, then to Barkot, where one road branches to Gangotri. The road winds along the Yamuna river through luxurious dense green vegetation to Hanumanchatti, the end of motorable road. The remaining journey has to be undertaken on foot or pony. Yamunotri is only 13 km from Hanumanchatti. But it is better to proceed another 6 km and have the night halt at Janakibaichatti. The journey to Yamunotri is simply breathtaking. High snowcovered peaks all around, glaciers, streams and waterfalls, vibrant green foliage and the pristine air are a sheer delight to tired city lungs. Yamunotri, 3322 metres above sea level, is located on the western bank of the great peak of Banderpunch (monkey's tail) which is 6315 metres high.	
	(i)	Questions: State the reason of taking halt at Mussouri. Ans: The reason of taking halt at Mussouri was taking breakfast there.	01
	(ii)	Mention the reason to undertake remaining journey to Yamunotri on foot or pony. Ans: The road that winds along the Yamuna river through dense green vegetation to Hanumanchatti was the end of motorable road. This is the reason for undertaking remaining journey to Yamunotri on foot or pony.	01



Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	iii)	Explain why the journey to Yamunotri is simply breathtaking. Ans: High snowcovered peaks all around, glaciers, streams and waterfalls, vibrant green foliage and the pristine air are a sheer delight to tired city lungs. So the journey to Yamunotri is simply breathtaking.	02
	iv)	Describe the location of Yamunotri. Ans: Yamunotri is 13 km from Hanumanchatti. It is 3322 metres above sea level and located on the western bank of the great peak of Banderpunch (monkey's tail) which is 6315 metres high.	02
	(C) (i)	Do as directed: Mary (watch) Television now as she (complete) her work. (Rewrite the sentence using the correct form of the verbs given in the brackets) Ans: Mary <u>is watching</u> Television now as she <u>has completed</u> her work.	02
	(ii)	Don't pluck flowers. (Change into Assertive) Ans: You are ordered not to pluck flowers.	02
	(iii)	It is a very beautiful city. (Change into Exclamatory) Ans: What a beautiful city it is!	02



SUMMER– 2019 EXAMINATION

Subject Name: English

Model Answer

Subject Code: 22101

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions, credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	(a)	Attempt any FIVE of the following: State any four qualities of Shiva that have inspired you. Ans: Shiva's qualities that have inspired me are his intelligence, hardworking nature, dedication, obedience and being a visionary. (Note: Any four correct qualities should be given marks.)	(10) 02
	(b)	Express your views on Cristiano Ronaldo. Ans: Cristiano Ronaldo is a Real Madrid star and Portugal football team's captain. He was born in a very poor family. His mother was a cook and his father was a kit man in a local football club. He used to practice daily from morning to night. Ronaldo had to go away from his family as he got an offer for better training. He was depressed and home sick. He used to call home and cry daily. He wanted to quit but his determination was stronger than his self-doubt and focused all his energy in improving his game. Cristiano Ronaldo got selected for Portugal's most famous club "Sporting Lisbon" and even got selected for the top 11 in the Euro 2004 squad for Portugal at the age of 19. Cristiano is the most decorated superstar and legend to ever play the sport.	02



Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	(c)	Name the scientists and lawyers who flourished under the leadership of JRD Tata. Ans: The scientists of international repute such as Homi Bhabha and leading lawyers such as J. D. Choksi and Nani Palkhivala flourished under JRD Tata's environment.	02
	(d)	State any three adjectives to describe the condition of patients in the abandoned ward in Patna. Ans: The adjectives to describe the condition of patients in the abandoned ward in Patna are as below: 1. Abandoned 2. Lawaris 3. Sick 4. Underprivileged (Note: Any three correct adjectives should be given marks.)	02
	(e)	State any three health hazards of improper disposal of e-waste. Ans : Health hazards of improper disposal of e-waste are as follows: 1. Due to improper disposal of e-waste, hazardous chemicals like lead, cadmium, beryllium, mercury and brominated flame retardants pollute the air at a high risk. 2. It contaminates soil and leaching into water sources. 3. If e-waste goes in landfill water, it introduces lethal toxicity which causes health risks range from kidney disease and brain damage to genetic mutations. (Note: Any other three correct points should be given marks.)	02
	(f)	Express in two sentences about the importance of scoring marks for your career. Ans: Scoring good marks is a pedestal for the career and our future growth. It helps in getting thorough knowledge which is necessary to get admission in a good and reputed institute and getting the job as well. (Note: Contents may vary.)	02



	(g) Name the peaks climbed by Arunima Sinha. Ans. : Arunima Sinha climbed following peaks: 1. Mount Everest in Asia 2. Mount Kilimanjaro in Africa 3. Mount Elbrus in Europe 4. Mount Kosciuszko in Australia 5. Mount Aconcagua in South America 6. Mount Carstensz (Note: Any four correct names of peaks should be given marks.)	½ ½ ½ ½ ½ ½
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		understand the entire issue in a minute to a major technical glitch and gave simple and short instructions. <i>(Note: Any other four correct qualities should be given marks.)</i>	01
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SLOT



Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	(c)	Describe the role and responsibilities of the society in controlling e-waste hazards. Ans: The society needs to take some measures to control e-waste. The roles and responsibilities are as follows: 1. We need to arrange public awareness programme like road show, T.V. show, appealing public by notices etc. on e-waste so that everybody will be aware about this serious issue. 2. Every time we replace one of our electronic devices, we must be sure to recycle the old one properly. 3. We can arrange e-waste recycling drive in main part of the cities and villages in collaboration with responsible and authentic e-waste recyclers. 4. We should appeal to donate old but working gadgets to the needy people. 5. We should reduce the consumption of electronic gadgets. 6. We should be a part of the solution by making Earth a better place by halting digital dumps. (Note: Any four correct points should be given marks.)	01 01 01 01 01 01



	(d)	Summarise the struggle of Shiva. Ans: Shiva with his elder sister Yogeshwari used to sell the garlands at traffic signals and other places after their school. Despite all the hard work his family members did, money was insufficient. Therefore Shiva used to sell the newspapers. Though Krishna paid Shiva's school fees, Shiva's family needed cash for rent and food among other things. So he continued working and studying hard. After his schooling, he got admission in Computer Science Engineering. He studied hard and scored good marks. Then he gave up their flower work. Now, after classes, Shiva began working late into the night as a salesman in an electronics shop. At one point, Shiva left his job at the electronics store and started a computer repair service with a friend. With all this, Shiva sometimes reported late for college. Being tired, he would take a nap in the class. Classmates giggled and teachers rebuked him. Meanwhile, from newspaper delivery boy in the mornings, Shiva graduated to sub agent with about 50 customers. His earnings increased substantially, but the workload had shot up too. He was still delivering papers but also had to visit homes and offices to collect payments and acquire more customers. On Krishna's suggestion, he also enrolled in a CAT coaching class. However after attending a few lectures, Shiva realized that he didn't have the time to attend the lectures. So he kept taking their notes and old CAT question papers to study at home. Krishna prepared him for mock interviews of IIM and group discussions. Together, they spent hours preparing for IIM admission. He got admission in IIM. After his IIM, he got a job as a deputy country manager in Sri Lanka.	04
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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	(a)	Attempt any THREE of the following : Develop a dialogue between a teacher and student who is apologising for using mobile phone in the class. (8-10 dialogues). Ans: Teacher : Sumeet! What are you doing? Sumeet : Nothing sir. Teacher : Then what is there in your hands? Sumeet : Sorry sir, I was just checking the message on my phone. Teacher: This is too much. You should not use mobile phone in the classroom. Sumeet : I apologise sir. I will not use the mobile phone in the class henceforth. Teacher : Technology should be used for our benefit but it should not overtake us. We should not misuse it. Sumeet : Sorry sir. I will take care of it. Teacher : Okay. Now pay attention to the study. Sumeet : Okay sir. <i>(Note: Contents may vary.)</i>	(12)
	b)	Develop a dialogue between Arjun and Amit discussing about the ways to improve their spoken English. (8-10 dialogues) Ans: Arjun: Hi Amit, How are you? Amit : I am great. What about you? Arjun : I am fine. What are you doing? Amit : I am reading article on the ways to improve spoken English. It is interesting. Arjun : That's good! Apart from reading articles, listening can also help in improving spoken English. Amit : Yes. It helped me also. Communicating in English also helps me to improve my spoken English. Arjun: Yes. I too use the same method to improve my spoken English. Amit : That's great. See you later. I have a class now. Arjun : Bye. See you. <i>(Note: Contents may vary.)</i>	04



	(c) Write a paragraph on “an online class compared to a traditional face-to face class”. (75 words) Ans: There are two types of classes - Traditional face-to-face class and Online class. If we compare both, the traditional classes are better than online classes in many ways. First, in the traditional classes, a personal attention can be given to the particular student whereas it is not possible in online classes because learner has to learn by himself/herself. Learning in a class with same age group is fun and interesting thing for the students. It is also helpful in memorizing which is possible only in a	
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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.		traditional class. Whereas there are no such types of interactive groups in online classes. This can lead to monotonousness. In online classes, computer or mobile may affect our eye sight but there is no such harm in traditional classes. In traditional classes, moral values are taught whereas in online classes, the importance is given only to curriculum. In traditional class, every doubt of a child can be cleared whereas it is not possible in online classes. Traditional classes can clear the concept of a child in a manner which he/she understands but for online classes, the content is fixed and cannot be changed according to the learners' ability. (Note: Contents may vary.)	04
	(d)	Write a paragraph on "Wi-Fi Technology". (75 words) Ans: Wi-Fi is a technology that uses radio waves to provide network connectivity. It stands for wireless frequency. It commonly uses the 2.4 gigahertz UHF and 5 gigahertz SHF ISM radio bands. With the help of Wi-Fi, we can have a very fast access to internet which can be used for service like checking an email, downloading as well as watching songs and movies, checking updates etc. Wi-Fi service is provided free of cost by governments at some places like the railway stations or bus stations. In order to connect to a Wi-Fi, a device has to be equipped with a wireless interface connection. Its range depends on Wi-Fi router. Wi-Fi can be used in Mobile phones, laptops, personal computer, tablets, smart TVs, gaming consoles etc. (Note: Contents may vary.)	04
4.		Attempt any THREE of the following :	(12)
	(a)	Make sentences using the following collocation:	
	(i)	Do your best Ans: My coach told me to do my best in the next competitions.	01
	(ii)	Break a record Ans: Virat will break a record of fastest century in ODI cricket.	01
	(iii)	Take an exam Ans: Why don't you take an exam?	01
	(iv)	Save electricity Ans: It is our moral duty to save electricity. (Note: Sentences may vary.)	01
	(b)	Replace the underlined words with correct word and rewrite the sentences.	
	(i)	Amit's father is very <u>industrial</u>. Ans: Amit's father is very <u>industrious</u>.	01



	(ii)	A <u>hoard</u> of dacoits looted the village. Ans: A <u>herd</u> of dacoits looted the village.	01
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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	(iii)	The goods train is <u>stationery</u> . Ans: The goods train is <u>stationary</u> .	01
	(iv)	Raju is very <u>week</u> . Ans: Raju is very <u>weak</u> .	01
	(c)	Write the meanings of the following idioms and make sentences of your own:	
	(i)	Blessing in disguise Ans: Meaning: something good which is not identified at first. Example: He was not happy with the offer but it was actually a blessing in disguise.	01 01
	(ii)	Crocodile tears Ans: Meaning: tears or expressions of sorrow those are insincere /false Example : She shed crocodile tears over her relative's death. (Note: Examples may vary.)	01 01
	(d)	Choose the correctly spelt word from the following :	
	(i)	unique, unic, unque Ans: unique	01
	(ii)	counsellor, counselar, counselor Ans: counselor	01
	(iii)	angrily, angryly, angrilly Ans: angrily	01
	(iv)	Keys, Keyes, Keies Ans: Keys	01
	(e)	Change the voice :	
	(i)	Let the window be shut. Ans: Shut the window.	01
	(ii)	He will be forgiven by the teacher. Ans: The teacher will forgive him.	01
	(iii)	Will your brother pay your fees today? Ans: Will your fees be paid by your brother today?	01
	(iv)	The robber has been caught by the police.	



Ans: The police have caught the robbers.

01

STO

04



	(c) Prepare a vote of thanks for the annual gathering of your institute. Ans : Good afternoon ladies and gentlemen. I am standing before you to propose vote of thanks. At the outset, I take immense pleasure in extending my sincere thanks to our honorable Founder President Dr. R. D. Patil for giving us an opportunity to organize Annual Gathering. I am also thankful to our principal Dr. S. G. Pathak who motivated and	
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Q. No.	Sub Q. N.	Answer	Marking Scheme
5		supported us for the success of this gathering. I thank the chief guest Mr. S. D. Thakur from the bottom of my heart for taking out some time for us and gracing the event by his presence. I express my heartfelt thanks to all the teachers who inspired us during this Annual Gathering. I thank all the organisers, event coordinators, technicians and my colleagues; without their support the event would not have been successful. I also want to thank our mess incharge for providing us the tasty food and tea. Last but not the least, I thank all the participants and students who have joined us today. Thank you all. (Note: Contents may vary.)	04
	(B)	Attempt any TWO : Change into indirect speech:	(04)
	(a)	The monitor said, "Sir, some students were making noise yesterday". Ans: The monitor told sir/the teacher that some students had been making noise the previous day/ the day before.	02
	(b)	The Prime Minister said. "India became independent in 1947". Ans: The Prime Minister said that India became independent in 1947.	02
	(c)	"Sit down." She told him. Ans: She told him to sit down.	02
6.		Attempt any TWO of the following :	(12)
	(a)	Do as directed :	
	(i)	Not everyone gets <u>the</u> opportunity to become <u>an</u> famous person. (Rewrite the sentence using correct article in place of the underlined article.) Ans: Not everyone gets <u>an</u> opportunity to become <u>a</u> famous person.	02
	(ii)	<u>Either</u> the police officers <u>or</u> the military personnel were present there (Rewrite the sentence using correct conjunctions in place of underlined conjunction). Ans: <u>Neither</u> the police officers <u>nor</u> the military personnel were present there.	
		OR	
		<u>Not only</u> the police officers <u>but also</u> the military personnel were present there.	02
	(iii)	Mr. Verma received a parcel <u>to</u> his son who had been staying in Mysore <u>from</u> many years. (Rewrite the sentence using the correct prepositions in place of underlined preposition.) Ans: Mr. Verma received a parcel <u>from</u> his son who had been staying in	



		Mysore for many years.	02
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SLOT

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	Read the following passage and answer the given questions : Our forests are important source of natural wealth covering one fifth of the country's land area. They supply the timber for the railways, house building and furniture making. They also supply the fuel for cooking and bamboo for paper making. Medicinal herbs also come from the forests. In the olden days our wise men and philosophers had their "Ashrams" in the forests. These Ashrams were centers of knowledge and learning. Unfortunately forests were ruthlessly cut down in the past few years. While the forests are disappearing the demand for timber is growing. We must therefore protect and develop our forests. "Vana Mahotsava" is a most important step in this direction. It is observed for a week in July every year. During this week lakhs of saplings are planted throughout the country. This is being done to restore our fast disappearing forests. Questions :	
	(i)	What do forests supply for paper making ? Ans: Forests supply bamboo for paper making.	01
	(ii)	What was the purpose of Ashrams? Ans: The purpose of Ashrams was to provide knowledge and learning.	01
	(iii)	Why is Vana Mahotsava observed? How is it observed? Ans: Vana Mahotsava is observed in order to protect, develop and restore our fast disappearing forests. Vana Mahotsava is observed for a week in July, when lakhs of saplings are planted throughout the country.	02
	(iv)	What would you do to restore forests? Ans: I would do following things to restore forests: 1. I would plant trees. 2. I would motivate others to plant trees. 3. I would take care of existing trees. <i>(Note: Points may vary. Any two correct points should be given marks.)</i>	01 01 01
	(C)	Do as directed:	
	(i)	Yesterday evening the phone (ring) three times while we (have) dinner. (Rewrite the sentence using the correct forms of the verbs in the brackets). Ans: Yesterday evening the phone <u>rang</u> three times while we <u>were having</u> dinner.	02
	(ii)	While I (do) the language course, I (meet) lots of people from all over the world. (Rewrite the sentence using the correct forms of the verbs in the brackets). Ans: While I <u>was doing</u> the language course, I <u>met</u> lots of people from all over the world.	02
	(iii)	What a wonderful painting this is ! (Change into Assertive) Ans: This is a very wonderful painting.	02

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- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
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- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		Attempt any FIVE of the following:	10
	a)	Prove that $\frac{1}{\log_3 6} + \frac{1}{\log_8 6} + \frac{1}{\log_9 6} = 3$	02
	Ans	$L.H.S = \frac{1}{\log_3 6} + \frac{1}{\log_8 6} + \frac{1}{\log_9 6}$ $= \frac{\log 3}{\log 6} + \frac{\log 8}{\log 6} + \frac{\log 9}{\log 6}$ $= \frac{\log (3 \times 8 \times 9)}{\log 6}$ $= \frac{\log 216}{\log 6}$ $= \frac{\log 6^3}{\log 6}$ $= \frac{3 \log 6}{\log 6}$ $= 3 = R.H.S$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

b)

Find x , if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$

SLOT

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b)	$\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$ $4(-2x - 28) - 3(3x - 77) + 9(12 + 22) = 0$ $\therefore -8x - 112 - 9x + 231 + 306 = 0$ $\therefore -17x + 425 = 0$ $\therefore x = 25$	$\frac{1}{2}$ $\frac{1}{2}$ 1
	c)	Without using calculator, find the value of $\cos(105^\circ)$ $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$ $= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{1 - \sqrt{3}}{2\sqrt{2}} \text{ or } -0.2588$	02 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	d)	The area of a rectangular garden is 3000 m^2. Its sides are in the ratio 6:5. Find the perimeter of the garden $\therefore$ Sides are in the ratio 6:5 $\therefore$ length = $6x$, breadth = $5x$ Area = $(6x)(5x)$ $3000 = 30x^2$ $\therefore x^2 = 100$ $\therefore x = 10$ $\therefore$ Length = 60 m, Breadth = 50 m Perimeter = $2(\text{length} + \text{breadth})$ $= 2(60 + 50) = 220$	02 $\frac{1}{2}$ 1
	e)	Find the area of ring between two concentric circles whose circumferences are 75cm and 55 cm.	02
	Ans	Area of ring = $A(\text{larger circle}) - A(\text{smaller circle})$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	e)	$\therefore \text{Area of ring} = \pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$ $\therefore 2\pi r_1 = 75$ $\therefore r_1 = \frac{75}{2\pi}$ $\therefore 2\pi r_2 = 55$ $\therefore r_2 = \frac{55}{2\pi}$ $\text{Area of ring} = \pi (r_1^2 - r_2^2)$ $= \pi \left(\left(\frac{75}{2\pi} \right)^2 - \left(\frac{55}{2\pi} \right)^2 \right)$ $= 206.9$	½
	f)	Find the range and coefficient of range 40, 52, 47, 28, 45, 36, 47, 50	02
	Ans	Range = $L - S$ $= 52 - 28$ $= 24$ Coefficient of range = $\frac{L - S}{L + S}$ $= \frac{52 - 28}{52 + 28}$ $= 0.3$	1
	g)	The two sets of observations are given below: Set I Set II $\bar{x} = 82.5$ $\bar{x} = 48.75$ $\sigma = 7.3$ $\sigma = 8.35$ Which of two sets is more consistent?	02
	Ans	For Set I $C.V = \frac{\sigma}{\bar{x}} \times 100$	1

[illegible]



Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	b)	If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$, find $A^2 - 8A$. Ans $A^2 = AA = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ $A^2 = \begin{bmatrix} 4+16+16 & 8+8+16 & 8+16+8 \\ 8+8+16 & 16+4+16 & 16+8+8 \\ 8+16+8 & 16+8+8 & 16+16+4 \end{bmatrix}$ $A^2 = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix}$ $8A = 8 \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix}$ $\therefore A^2 - 8A = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ c) Resolve into partial fractions $\frac{3x+2}{(x+1)(x^2-1)}$ Ans $\frac{3x+2}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$ $\therefore 3x+2 = A(x-1)(x+1) + B(x-1) + C(x+1)^2$ Put $x = -1$ $\therefore -3+2 = B(-1-1)$ $\boxed{B = \frac{1}{2}}$ Put $x = 1$	04 1 1 1 1 04 ½ 1



Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	c)	$\therefore 3+2 = C(1+1)^2$ $\boxed{C = -\frac{5}{4}}$ Put $x = 0, B = \frac{1}{2}, C = \frac{5}{4}$ $\therefore 2 = A(0-1)(0+1) + \frac{1}{2}(0-1) + \frac{5}{4}(0+1)^2$ $\boxed{A = -\frac{5}{4}}$ $\therefore \frac{3x+2}{(x+1)^2(x-1)} = \frac{-\frac{5}{4}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2} + \frac{\frac{5}{4}}{x-1}$	1
	d)	A metal strip having sides $17 \times 7 \times 5$ cm is melted down and minted into coins each of diameter 1.4 cm and thickness 0.08 cm. Assuming no wastage, how many coins can be minted? Ans Metal strip has dimensions $17 \times 7 \times 5$ cm Volume of metal strip $= 17 \times 7 \times 5 = 595 \text{ cm}^3$ Coin has diameter 1.4 cm $\therefore$ radius $= 0.7 \text{ cm}$ Thickness of coin $= 0.08 \text{ cm}$ Volume of one coin $= \pi r^2 h$ $= \pi (0.7)^2 (0.08)$ $= 0.123$ Number of coin minted $= \frac{\text{Volume of metal strip}}{\text{Volume of one coin}}$ $= \frac{595}{0.123}$ $= 4837.4 \approx 4837$	1
			$\frac{1}{2}$
			$\frac{1}{2}$
			04
			1
			2
3.		Attempt any THREE of the following:	12
	a)	Prove that $\tan 70^\circ - \tan 50^\circ - \tan 20^\circ = \tan 70^\circ \tan 50^\circ \tan 20^\circ$	04
	Ans	$\therefore 70^\circ - 20^\circ = 50^\circ$	



Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	$\tan(70^\circ - 20^\circ) = \tan 50^\circ$ $\frac{\tan 70^\circ - \tan 20^\circ}{1 + \tan 70^\circ \tan 20^\circ} = \tan 50^\circ$ $\tan 70^\circ - \tan 20^\circ = \tan 50^\circ (1 + \tan 70^\circ \tan 20^\circ)$ $\tan 70^\circ - \tan 20^\circ = \tan 50^\circ + \tan 50^\circ \tan 70^\circ \tan 20^\circ$ $\tan 70^\circ - \tan 50^\circ - \tan 20^\circ = \tan 70^\circ \tan 50^\circ \tan 20^\circ$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
	b)	Prove that $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \left(\frac{\theta}{2} \right)$	04
	Ans	$\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta}$ $= \frac{1 - \cos \theta + \sin \theta}{1 + \cos \theta + \sin \theta}$ $= \frac{2 \sin^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}}$ $= \frac{2 \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2} \right)}{2 \cos \frac{\theta}{2} \left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2} \right)}$ $= \tan \left(\frac{\theta}{2} \right)$	2 1 1
	c)	Prove that $\frac{\cos 2A + 2 \cos 4A + \cos 6A}{\cos A + 2 \cos 3A + \cos 5A} = \cos A - \sin A \tan 3A$	04
	Ans	$\frac{\cos 2A + 2 \cos 4A + \cos 6A}{\cos A + 2 \cos 3A + \cos 5A} = \frac{2 \cos 4A + \cos 2A + \cos 6A}{2 \cos 3A + \cos A + \cos 5A}$ $= \frac{2 \cos 4A + 2 \cos \left(\frac{2A+6A}{2} \right) \cos \left(\frac{2A-6A}{2} \right)}{2 \cos 3A + 2 \cos \left(\frac{A+5A}{2} \right) \cos \left(\frac{A-5A}{2} \right)}$	1



Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\frac{\cos 2A + 2 \cos 4A + \cos 6A}{\cos A + 2 \cos 3A + \cos 5A} = \frac{2 \cos 4A + 2 \cos 4A \cos(-2A)}{2 \cos 3A + 2 \cos 3A \cos(-2A)}$ $= \frac{2 \cos 4A (1 + \cos(-2A))}{2 \cos 3A (1 + \cos(-2A))}$ $= \frac{\cos 4A}{\cos 3A}$ $= \frac{\cos(3A + A)}{\cos 3A}$ $= \frac{\cos 3A \cos A - \sin 3A \sin A}{\cos 3A}$ $= \frac{\cos 3A \cos A}{\cos 3A} - \frac{\sin 3A \sin A}{\cos 3A}$ $= \cos A - \tan 3A \sin A$ $= R.H.S$	1/2 1/2 1/2 1/2 1
	d)	Prove that $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$	04
	Ans	$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$ $= \frac{\sqrt{3}}{2} [\sin 40^\circ \sin 80^\circ] \sin 20^\circ$ $= \frac{\sqrt{3}}{4} [\cos 40^\circ - \cos 120^\circ] \sin 20^\circ$ $= \frac{\sqrt{3}}{4} [\cos 40^\circ - \cos(180^\circ - 60^\circ)] \sin 20^\circ$ $= \frac{\sqrt{3}}{4} [\cos 40^\circ + \cos 60^\circ] \sin 20^\circ$ $= \frac{\sqrt{3}}{4} \left[\cos 40^\circ + \frac{1}{2} \right] \sin 20^\circ$ $= \frac{\sqrt{3}}{4} \left[\cos 40^\circ \sin 20^\circ + \frac{1}{2} \sin 20^\circ \right]$	1/2 1/2 1/2 1/2 1/2 1/2



Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	Resolve in to partial fractions $\frac{x^4}{x^3+1}$ Ans $\begin{array}{r} x^3+1 \overline{) x^4} \\ x^4 + x \\ \hline -x \end{array}$ $\frac{x^4}{x^3+1} = x - \frac{x}{x^3+1}$ $\frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ $\therefore x = (x^2-x+1)A + (x+1)(Bx+C)$ Put $x = -1$ $\therefore -1 = 3A$ $\therefore A = \frac{-1}{3}$ Put $x = 0$ $0 = (1)A + (1)C$ $0 = \frac{-1}{3} + C$ $\therefore C = \frac{1}{3}$ Put $x = 1$ $\therefore 1 = (1)A + 2(B+C)$ $\therefore 1 = \frac{-1}{3} + 2B + \frac{2}{3}$ $\therefore 1 - \frac{1}{3} = 2B$ $\therefore \frac{2}{3} = 2B$	04 $\frac{1}{2}$ 1 1



Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	$\therefore B = \frac{1}{3}$ $\therefore \frac{x}{(x+1)(x^2-x+1)} = \frac{-1}{3} + \frac{1}{3} \frac{x+1}{x^2-x+1}$ $\frac{x^4}{x^3+1} = -\frac{1}{3} + \frac{1}{3} \frac{x+1}{x^2-x+1}$	1
	c)	Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$ Ans $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ $= \pi + \tan^{-1}\left(\frac{1+2}{1-(1)(2)}\right) + \tan^{-1}(3)$ $= \pi + \tan^{-1}(-3) + \tan^{-1}(3)$ $= \pi - \tan^{-1}(3) + \tan^{-1}(3)$ $= \pi$	1/2
	d)	Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ Ans Let $\sin^{-1}\left(\frac{3}{5}\right) = A$ $\therefore \sin A = \frac{3}{5}$ $\therefore \cos^2 A = 1 - \sin^2 A$ $= 1 - \frac{9}{25}$ $= \frac{16}{25}$ $\therefore \cos A = \frac{4}{5}$	04
			1



Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		$\sin^{-1}\left(\frac{8}{17}\right) = B \quad \therefore \sin B = \frac{8}{17}$ $\therefore \cos^2 B = 1 - \sin^2 B$ $= 1 - \frac{64}{289}$ $= \frac{225}{289}$ $\therefore \cos B = \frac{15}{17}$ $\therefore \cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \frac{4}{5} \times \frac{15}{17} + \frac{3}{5} \times \frac{8}{17}$ $\therefore \cos(A - B) = \frac{84}{85}$ $\therefore A - B = \cos^{-1}\left(\frac{84}{85}\right)$ $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ e) Without using calculator, Prove that $\sin 420^\circ \cos 390^\circ + \cos(-300^\circ) \sin(-330^\circ) = 1$ Ans $\sin 420^\circ = \sin(90^\circ \times 4 + 60^\circ)$ $= \sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 390^\circ = \cos(90^\circ \times 4 + 30^\circ)$ $= \cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos(-300^\circ) = \cos(300^\circ)$ $= \cos(90^\circ \times 3 + 30^\circ)$ $= \sin 30^\circ = \frac{1}{2}$	1 1 1 04 ½ ½ ½ ½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\sin(-330^\circ) = -\sin(330^\circ)$ $= -\sin(90^\circ \times 3 + 60^\circ)$ $= -(-\cos 60^\circ) = \frac{1}{2}$ $\sin 420^\circ \cos 390^\circ + \cos(-300^\circ) \sin(-330^\circ)$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$ $= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
5.		Attempt any TWO of the following: a) Attempt the following: i) Find the acute angle between the lines $y = 5x + 6$ and $y = x$. Ans For $y = 5x + 6 \therefore 5x - y + 6 = 0$ slope $m_1 = -\frac{a}{b} = -\frac{5}{-1} = 5$ For $y = x \therefore x - y = 0$ slope $m_2 = -\frac{a}{b} = -\frac{1}{-1} = 1$ $\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right$ $= \left \frac{5 - 1}{1 + 5 \times 1} \right$ $\therefore \tan \theta = \frac{2}{3}$ $\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right)$	12 06 03 1 1 1
	ii)	Find the equation of the line passing through the point $(4, 5)$ and perpendicular to the line $7x - 5y = 420$.	03



Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)ii)	Point $= (x_1, y_1) = (4, 5)$	
	Ans	Slope of the line $7x - 5y = 420$ is, $m = -\frac{a}{b} = -\frac{7}{-5} = \frac{7}{5}$ $\therefore$ Slope of the required line is, $m = -\frac{1}{\frac{7}{5}} = -\frac{5}{7}$ $\therefore$ equation is, $y - y_1 = m_1(x - x_1)$ $\therefore y - 5 = -\frac{5}{7}(x - 4)$ $\therefore 5x + 7y - 55 = 0$	1
	b)	Attempt the following:	
	i)	Find the length of the perpendicular from the point $(2, 3)$ on the line $4x - 6y - 3 = 0$.	
	Ans	$p = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ $= \frac{ 4(2) + (-6)(3) - 3 }{\sqrt{(4)^2 + (-6)^2}}$ $= \frac{ 8 - 18 - 3 }{\sqrt{52}}$ $= \frac{13}{\sqrt{52}} \text{ or } 1.803$	1
	ii)	Find the equation of the line passing through $(1, 7)$ and having slope 2 units.	
	Ans	Point $= (x_1, y_1) = (1, 7)$ & slope $= 2$ $\therefore$ Equation of line is, $y - y_1 = m(x - x_1)$ $\therefore y - 7 = 2(x - 1)$ $\therefore 2x - y + 5 = 0$	03
			1
			2



Q. No.	Sub Q.N.	Answers	Marking Scheme												
5.	c)	Attempt the following:	06												
	i)	A square grassy plot is of side 100 meters.It has a gravel path 10 meters wide all round it on the inside.Find the area of the path.	03												
	Ans	Area of path = Area of grassy plot – Area of inner gravel path $= (100)^2 - (80)^2$ $= 3600 \text{ sq.m.}$	3												
	ii)	The volume of a sphere is $\frac{88}{21}$ cubic meters. Find its surface area.	03												
	Ans	Volume of sphere = $\frac{4}{3} \pi r^3$ $\therefore \frac{4}{3} \pi r^3 = \frac{88}{21}$ $r^3 = \frac{88}{21} \times \frac{3}{4} \times \frac{7}{22}$ $r^3 = 1$ $r = 1$ Surface area of sphere = $4\pi r^2$ $= 4\pi (1)^2 = 4\pi \text{ sq.m.}$	1												
6.	Attempt any TWO of the following:		12												
	a)(i)	Find the mean deviation from mean of the following distribution:	03												
		<table><tr><th>C.I</th><th>0-10</th><th>10-20</th><th>20-30</th><th>30-40</th><th>40-50</th></tr><tr><td>f_i</td><td>5</td><td>8</td><td>15</td><td>16</td><td>6</td></tr></table>	C.I	0-10	10-20	20-30	30-40	40-50	f_i	5	8	15	16	6	
C.I	0-10	10-20	20-30	30-40	40-50										
f_i	5	8	15	16	6										

Q. No.	Sub Q.N.	Answers	Marking Scheme																																										
6.	a)(i)																																												
	Ans	<table><tr><th>C.I.</th><th>f_i</th><th>x_i</th><th>$f_i x_i$</th><th>$d_i = x_i - \bar{x}$</th><th>$f_i d_i$</th></tr><tr><td>0-10</td><td>5</td><td>5</td><td>25</td><td>22</td><td>110</td></tr><tr><td>10-20</td><td>8</td><td>15</td><td>120</td><td>12</td><td>96</td></tr><tr><td>20-30</td><td>15</td><td>25</td><td>375</td><td>2</td><td>30</td></tr><tr><td>30-40</td><td>16</td><td>35</td><td>560</td><td>8</td><td>128</td></tr><tr><td>40-50</td><td>6</td><td>45</td><td>270</td><td>18</td><td>108</td></tr><tr><td></td><td>$\sum f_i = 50$</td><td></td><td>$\sum f_i x_i = 1350$</td><td></td><td>$\sum f_i d_i = 472$</td></tr></table>	C.I.	f_i	x_i	$f_i x_i$	$d_i = x_i - \bar{x} $	$f_i d_i$	0-10	5	5	25	22	110	10-20	8	15	120	12	96	20-30	15	25	375	2	30	30-40	16	35	560	8	128	40-50	6	45	270	18	108		$\sum f_i = 50$		$\sum f_i x_i = 1350$		$\sum f_i d_i = 472$	1
	C.I.	f_i	x_i	$f_i x_i$	$d_i = x_i - \bar{x} $	$f_i d_i$																																							
	0-10	5	5	25	22	110																																							
	10-20	8	15	120	12	96																																							
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	40-50	6	45	270	18	108																																							
		$\sum f_i = 50$		$\sum f_i x_i = 1350$		$\sum f_i d_i = 472$																																							
		$Mean\ x = \frac{\sum f_i x_i}{\sum f_i}$																																											
	$\therefore \bar{x} = \frac{1350}{50}$																																												
	$\therefore \bar{x} = 27$		1																																										
	$M.D = \frac{\sum f_i d_i}{\sum f_i}$																																												
	$\therefore M.D = \frac{472}{50}$																																												
	$\therefore M.D. = 9.44$		1																																										
	ii)	Find range & coefficient of range for the following data:	03																																										
		<table><tr><th>C.I</th><th>10-19</th><th>20-29</th><th>30-39</th><th>40-49</th><th>50-59</th></tr><tr><th>f</th><td>15</td><td>25</td><td>13</td><td>17</td><td>10</td></tr></table>	C.I	10-19	20-29	30-39	40-49	50-59	f	15	25	13	17	10																															
C.I	10-19	20-29	30-39	40-49	50-59																																								
f	15	25	13	17	10																																								



Q. No.	Sub Q.N.	Answers						Marking Scheme																																																								
6.	ii)	C.I	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5	1																																																								
	Ans	f	15	25	13	17	10																																																									
	Range = L – S						1																																																									
	= 59.5 – 9.5																																																															
	= 50																																																															
Coefficient of range = $\frac{L - S}{L + S}$						1																																																										
= $\frac{59.5 - 9.5}{59.5 + 9.5}$																																																																
= 0.725																																																																
	b)	Calculate standard deviation and co-efficient of variance of the following table:						06																																																								
		<table><tr><td>Marks below</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>No.of Students</td><td>6</td><td>16</td><td>28</td><td>38</td><td>46</td></tr></table>							Marks below	5	10	15	20	25	No.of Students	6	16	28	38	46																																												
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	Ans	<table><tr><th>Class</th><th>x_i</th><th>f_i</th><th>$f_i x_i$</th><th>$d_i = \frac{x_i - a}{h}$</th><th>$f_i d_i$</th><th>d_i^2</th><th>$f_i d_i^2$</th></tr><tr><td>0-5</td><td>2.5</td><td>6</td><td>15</td><td>-2</td><td>-12</td><td>4</td><td>24</td></tr><tr><td>5-10</td><td>7.5</td><td>10</td><td>75</td><td>-1</td><td>-10</td><td>1</td><td>10</td></tr><tr><td>10-15</td><td>12.5</td><td>12</td><td>150</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>15-20</td><td>17.5</td><td>10</td><td>175</td><td>1</td><td>10</td><td>1</td><td>10</td></tr><tr><td>20-25</td><td>22.5</td><td>8</td><td>180</td><td>2</td><td>16</td><td>4</td><td>32</td></tr><tr><td></td><td></td><td>46</td><td>595</td><td></td><td>4</td><td></td><td>76</td></tr></table>						Class	x_i	f_i	$f_i x_i$	$d_i = \frac{x_i - a}{h}$	$f_i d_i$	d_i^2	$f_i d_i^2$	0-5	2.5	6	15	-2	-12	4	24	5-10	7.5	10	75	-1	-10	1	10	10-15	12.5	12	150	0	0	0	0	15-20	17.5	10	175	1	10	1	10	20-25	22.5	8	180	2	16	4	32			46	595		4		76	3
Class	x_i	f_i	$f_i x_i$	$d_i = \frac{x_i - a}{h}$	$f_i d_i$	d_i^2	$f_i d_i^2$																																																									
0-5	2.5	6	15	-2	-12	4	24																																																									
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20-25	22.5	8	180	2	16	4	32																																																									
		46	595		4		76																																																									
		Mean $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{595}{46} = 12.935$																																																														
		$S.D. = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$						1																																																								



Q. No.	Sub Q. N.	Answers	Marking Scheme																																										
6.	b)	$S.D. = \sigma = \sqrt{\frac{76}{46} - \left(\frac{4}{46}\right)^2} \times 5$$S.D. = \sigma = 6.412$$\text{Coefficient of variance} = \frac{\sigma}{\bar{x}} \times 100$$= \frac{6.412}{12.935} \times 100$$= 49.57$ <u>OR</u> <table><thead><tr><th>Class Interval</th><th>x_i</th><th>f_i</th><th>$f_i x_i$</th><th>x_i^2</th><th>$f_i x_i^2$</th></tr></thead><tbody><tr><td>0-5</td><td>2.5</td><td>6</td><td>15</td><td>6.25</td><td>37.5</td></tr><tr><td>5-10</td><td>7.5</td><td>10</td><td>75</td><td>56.25</td><td>562.5</td></tr><tr><td>10-15</td><td>12.5</td><td>12</td><td>150</td><td>156.25</td><td>1875</td></tr><tr><td>15-20</td><td>17.5</td><td>10</td><td>175</td><td>306.25</td><td>3062.5</td></tr><tr><td>20-25</td><td>22.5</td><td>8</td><td>180</td><td>506.25</td><td>4050</td></tr><tr><td></td><td></td><td>46</td><td>595</td><td></td><td>9587.5</td></tr></tbody></table>$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{595}{46} = 12.935$$S.D. = \sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$$S.D. = \sigma = \sqrt{\frac{9587.5}{46} - (12.935)^2}$	Class Interval	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$	0-5	2.5	6	15	6.25	37.5	5-10	7.5	10	75	56.25	562.5	10-15	12.5	12	150	156.25	1875	15-20	17.5	10	175	306.25	3062.5	20-25	22.5	8	180	506.25	4050			46	595		9587.5	1 1 3 1
Class Interval	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$																																								
0-5	2.5	6	15	6.25	37.5																																								
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		46	595		9587.5																																								



Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	b)	S.D. = $\sigma = 6.412$ Coefficient of variance = $\frac{\sigma}{x} \times 100$ $= \frac{6.412}{12.935} \times 100$ $= 49.57$	1
	c)	Solve the following equations by matrix inversion method : $x + y + z = 6 \quad , \quad 3x - y + 3z = 10 \quad , \quad 5x + 5y - 4z = 3$	06
	Ans	Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 3 \\ 5 & 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $ A = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -1 & 3 \\ 5 & 5 & -4 \end{vmatrix}$ $ A = 1(4 - 15) - 1(-12 - 15) + 1(15 + 5)$ $\therefore A = 36 \neq 0$ $\therefore A^{-1} \text{ exists}$ Matrix of minors = $\begin{bmatrix} \begin{vmatrix} -1 & 3 \\ 5 & -4 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 5 & -4 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 5 & 5 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 5 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 5 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 5 & 5 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} \end{bmatrix}$ Matrix of minors = $\begin{bmatrix} -11 & -27 & 20 \\ -9 & -9 & 0 \\ 4 & 0 & -4 \end{bmatrix}$	1



Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	c)	Matrix of cofactors = $\begin{bmatrix} -11 & 27 & 20 \\ 9 & -9 & 0 \\ 4 & 0 & -4 \end{bmatrix}$ $\text{Adj.}A = \begin{bmatrix} -11 & 9 & 4 \\ 27 & -9 & 0 \\ 20 & 0 & -4 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{Adj.}A$ $A^{-1} = \frac{1}{36} \begin{bmatrix} -11 & 9 & 4 \\ 27 & -9 & 0 \\ 20 & 0 & -4 \end{bmatrix}$ $\therefore X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36} \begin{bmatrix} -11 & 9 & 4 \\ 27 & -9 & 0 \\ 20 & 0 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36} \begin{bmatrix} -66 + 90 + 12 \\ 162 - 90 + 0 \\ 120 + 0 - 12 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 36 \\ 72 \\ 108 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\therefore x = 1, y = 2, z = 3.$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
<u>Important Note</u> <i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i>			



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Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	a)	Attempt any five of the following :	10
	Ans	Evaluate $\log_3 81$	02
		$\log_3 81$	$\frac{1}{2}$
		$= \log_3 (3)^4$	$\frac{1}{2}$
		$= 4\log_3 3$	$\frac{1}{2}$
		$= 4(1)$	$\frac{1}{2}$
		$= 4$	$\frac{1}{2}$
		OR	
		$\log_3 81$	
		$= \frac{\log 81}{\log 3}$	$\frac{1}{2}$
		$= \frac{\log (3)^4}{\log 3}$	$\frac{1}{2}$
		$= \frac{4 \log 3}{\log 3}$	$\frac{1}{2}$
		$= 4$	$\frac{1}{2}$
		OR	
		Let $\log_3 81 = x$	
		$3^x = 81$	$\frac{1}{2}$
		$3^x = 3^4$	$\frac{1}{2}$
		$x = 4$	$\frac{1}{2}$
	b)	Find the area of the triangle whose vertices are $(4, 3)$, $(1, 4)$ and $(2, 3)$.	02
	Ans	Let $(x_1, y_1) = (4, 3)$, $(x_2, y_2) = (1, 4)$ and $(x_3, y_3) = (2, 3)$	



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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b)	$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $= \frac{1}{2} \begin{vmatrix} 4 & 3 & 1 \\ 1 & 4 & 1 \\ 2 & 3 & 1 \end{vmatrix}$ $= \frac{1}{2} [4(4-3) - 3(1-2) + 1(3-8)]$ $= 1$	1
	c)	Find the value of $\sin(15^\circ)$ using compound angles	02
	Ans	$\sin(15^\circ)$ $= \sin(45^\circ - 30^\circ)$ $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \text{or} \quad 0.2588$ OR $\sin(15^\circ)$ $= \sin(60^\circ - 45^\circ)$ $= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \text{or} \quad 0.2588$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	d)	Find the area of rhombus whose diagonals are 6 cm and 9 cm.	02
	Ans	$\text{Area of rhombus} = \frac{1}{2}(d_1 \times d_2)$ $= \frac{1}{2}(6 \times 9)$	1



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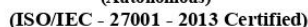
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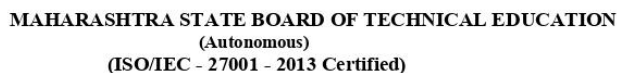
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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	d)	Area of rhombus = 27	1
	e)	The length , breadth and height of a cuboid are 8 cm,11 cm and 15 cm respectively.Find the total surface area.	02
	Ans	Let $l = 8$, $b = 11$, $h = 15$ Total surface Area of a cuboid $= 2[lb + bh + hl]$ $= 2[8 \times 11 + 11 \times 15 + 15 \times 8]$ $= 746$	1 1
	f)	Find the range of the data: 14 , 18 , 22 , 35 , 42 , 44 , 8 , 7 , 5 and 2	02
	Ans	Range $= L - S$ $= 44 - 2$ $= 42$	1 1
2.	g)	If mean is 34.5 and standard deviation is 5 find the coefficient of variance.	02
	Ans	Coefficient of variance $= \frac{\sigma}{x} \times 100$ $= \frac{5}{34.5} \times 100$ $= 14.493$	1 1
	a)	Attempt any three of the following:	12
		If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ prove that $A^2 = I$	04
	Ans	$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$	



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Q. No.	Sub Q. N.	Answers	Marking Scheme																												
2.	d)	Find mean of the following data:	04																												
		<table><tr><td>Class - Interval</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td></tr><tr><td>Frequency</td><td>3</td><td>5</td><td>8</td><td>3</td><td>1</td></tr></table>	Class - Interval	0-10	10-20	20-30	30-40	40-50	Frequency	3	5	8	3	1																	
	Class - Interval	0-10	10-20	20-30	30-40	40-50																									
	Frequency	3	5	8	3	1																									
	Ans	<table><tr><td>Class-Interval</td><td>x_i</td><td>f_i</td><td>$f_i x_i$</td></tr><tr><td>0-10</td><td>5</td><td>3</td><td>15</td></tr><tr><td>10-20</td><td>15</td><td>5</td><td>75</td></tr><tr><td>20-30</td><td>25</td><td>8</td><td>200</td></tr><tr><td>30-40</td><td>35</td><td>3</td><td>105</td></tr><tr><td>40-50</td><td>45</td><td>1</td><td>45</td></tr><tr><td></td><td></td><td>20</td><td>440</td></tr></table>	Class-Interval	x_i	f_i	$f_i x_i$	0-10	5	3	15	10-20	15	5	75	20-30	25	8	200	30-40	35	3	105	40-50	45	1	45			20	440	2
	Class-Interval	x_i	f_i	$f_i x_i$																											
	0-10	5	3	15																											
	10-20	15	5	75																											
	20-30	25	8	200																											
	30-40	35	3	105																											
40-50	45	1	45																												
		20	440																												
	Mean $\bar{x} = \frac{\sum f_i x_i}{N}$																														
	$\therefore \bar{x} = \frac{440}{20}$		1																												
	$\therefore \bar{x} = 22$		1																												
3.		Attempt any three of the following:	12																												
	a)	If $\tan A = \frac{1}{2}, \tan B = \frac{1}{3}$, find the value of $\tan(A + B)$	04																												
Ans		$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$																													
		$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$	2																												
		$= 1$	2																												



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3.	b)	Prove : $\tan\left(\frac{\pi}{4} + A\right) = \frac{\cos A + \sin A}{\cos A - \sin A}$ Ans $\tan\left(\frac{\pi}{4} + A\right)$ $= \frac{\tan \frac{\pi}{4} + \tan A}{1 - \tan \frac{\pi}{4} \tan A}$ $= \frac{1 + \tan A}{1 - \tan A}$ $= \frac{1 + \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}}$ $= \frac{\cos A + \sin A}{\cos A - \sin A}$ OR $\frac{\cos A + \sin A}{\cos A - \sin A}$ $= \frac{1 + \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}}$ $= \frac{1 + \tan A}{1 - \tan A}$ $= \frac{\tan \frac{\pi}{4} + \tan A}{1 - \tan \frac{\pi}{4} \tan A}$ $= \tan\left(\frac{\pi}{4} + A\right)$	04 1 1 1 1 1 1 1
	c)	Prove: $\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$	04



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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A}$ $= \frac{(\sin 4A + \sin 6A) + \sin 5A}{(\cos 4A + \cos 6A) + \cos 5A}$ $= \frac{2 \sin \left(\frac{4A + 6A}{2} \right) \cos \left(\frac{4A - 6A}{2} \right) + \sin 5A}{2 \cos \left(\frac{4A + 6A}{2} \right) \cos \left(\frac{4A - 6A}{2} \right) + \cos 5A}$ $= \frac{2 \sin 5A \cos(-A) + \sin 5A}{2 \cos 5A \cos(-A) + \cos 5A}$ $= \frac{\sin 5A [2 \cos(-A) + 1]}{\cos 5A [2 \cos(-A) + 1]}$ $= \tan 5A$	2 1 $\frac{1}{2}$ $\frac{1}{2}$
	d)	Prove : $\cos^{-1} \left(\frac{4}{5} \right) + \cos^{-1} \left(\frac{12}{13} \right) = \cos^{-1} \left(\frac{33}{65} \right)$	04
	Ans	Let $\cos^{-1} \left(\frac{4}{5} \right) = A$ $\therefore \cos A = \frac{4}{5}$ $\therefore \sin^2 A = 1 - \cos^2 A$ $= 1 - \frac{16}{25}$ $= \frac{9}{25}$ $\therefore \sin A = \frac{3}{5}$ $\cos^{-1} \left(\frac{12}{13} \right) = B$ $\therefore \cos B = \frac{12}{13}$ $\therefore \sin^2 B = 1 - \cos^2 B$ $\therefore \sin^2 B = 1 - \frac{144}{169}$	1



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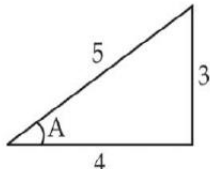
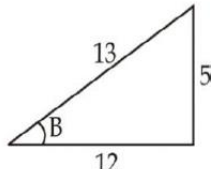
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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$\therefore \sin^2 B = \frac{25}{169}$ $\therefore \sin B = \frac{5}{13}$ $\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$ $= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$ $= \frac{48}{65} - \frac{15}{65}$ $\therefore \cos(A+B) = \frac{33}{65}$ $\therefore A+B = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$ <u>OR</u> Let $\cos^{-1}\left(\frac{4}{5}\right) = A$ $\therefore \cos A = \frac{4}{5}$ $\therefore \tan A = \frac{3}{4}$ $A = \tan^{-1}\left(\frac{3}{4}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ $\cos^{-1}\left(\frac{12}{13}\right) = B$ $\therefore \cos B = \frac{12}{13}$ $\therefore \tan B = \frac{5}{12}$  	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$B = \tan^{-1}\left(\frac{5}{12}\right)$ $\therefore \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$ $L.H.S. = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$ $= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)}\right)$ $= \tan^{-1}\left(\frac{56}{33}\right)$ Let $\tan^{-1}\left(\frac{56}{33}\right) = C$ $\therefore \tan C = \frac{56}{33}$ $\therefore \cos C = \frac{33}{65}$ $\therefore C = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	1 $\frac{1}{2}$ $\frac{1}{2}$ 1
4.		Attempt any three of the following: a) If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ show that $A^2 - 8A$ is scalar matrix. Ans $A^2 - 8A$ $= A.A - 8A$	12 04



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4.	a)	$= \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} - 8 \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ $= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix}$ $= \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ $\therefore A^2 - 8A$ is scalar matrix	2+1
	b)	Resolve into partial fraction: $\frac{3x-1}{(x-4)(x+1)(x-1)}$.	04
	Ans	$\frac{3x-1}{(x-4)(x+1)(x-1)} = \frac{A}{x-4} + \frac{B}{x+1} + \frac{C}{x-1}$ $\therefore 3x-1 = A(x+1)(x-1) + B(x-4)(x-1) + C(x-4)(x+1)$ Put $x = 4$ $3(4)-1 = A(4+1)(4-1)$ $\therefore 11 = 15A$ $\therefore A = \frac{11}{15}$ Put $x = -1$ $3(-1)-1 = B(-1-4)(-1-1)$ $\therefore -4 = B(-5)(-2)$ $\therefore B = \frac{-2}{5}$ Put $x = 1$ $3(1)-1 = C(1-4)(1+1)$ $\therefore 2 = C(-3)(2)$ $\therefore C = \frac{-1}{3}$	$\frac{1}{2}$ 1 1 1



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4.	b)	$\therefore \frac{3x-1}{(x-4)(x+1)(x-1)} = \frac{11}{x-4} + \frac{-2}{x+1} + \frac{-1}{x-1}$	½
	c)	Prove that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$	04
	Ans	$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{2} (2 \cos 20^\circ \cos 40^\circ) \cdot \left(\frac{1}{2}\right) \cos 80^\circ$ $= \frac{1}{4} [\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)] \cos 80^\circ$ $= \frac{1}{4} [\cos(60^\circ) + \cos(-20^\circ)] \cos 80^\circ$ $= \frac{1}{4} \left[\frac{1}{2} \cos 80^\circ + \cos 20^\circ \cos 80^\circ \right]$ $= \frac{1}{4} \left[\frac{1}{2} \cos 80^\circ + \frac{1}{2} (2 \cos 20^\circ \cos 80^\circ) \right]$ $= \frac{1}{8} [\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(20^\circ - 80^\circ)]$ $= \frac{1}{8} [\cos 80^\circ + \cos(100^\circ) + \cos(-60^\circ)]$ $= \frac{1}{8} \left[\cos 80^\circ + \cos(180 - 80^\circ) + \frac{1}{2} \right]$ $= \frac{1}{8} \left[\cos 80^\circ - \cos(80^\circ) + \frac{1}{2} \right]$ $= \frac{1}{16}$	½ ½ ½ ½ ½ ½ ½ ½
	d)	Prove: $\sin A \cdot \sin(60 - A) \cdot \sin(60 + A) = \frac{1}{4} \sin 3A$.	04
	Ans	$L.H.S. = \sin A \cdot \sin(60 - A) \cdot \sin(60 + A)$ $= \sin A (\sin 60 \cos A - \cos 60 \sin A) (\sin 60 \cos A + \cos 60 \sin A)$ $= \sin A \left[\frac{\sqrt{3}}{2} \cos A - \frac{1}{2} \sin A \right] \left[\frac{\sqrt{3}}{2} \cos A + \frac{1}{2} \sin A \right]$	½ 1



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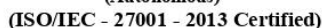
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4.	d)	$L.H.S. = \sin A \left[\left(\frac{\sqrt{3}}{2} \cos A \right)^2 - \left(\frac{1}{2} \sin A \right)^2 \right]$ $= \sin A \left[\frac{3}{4} \cos^2 A - \frac{1}{4} \sin^2 A \right]$ $= \frac{1}{4} \sin A [3 \cos^2 A - \sin^2 A]$ $= \frac{1}{4} \sin A [3 (1 - \sin^2 A) - \sin^2 A]$ $= \frac{1}{4} \sin A [3 - 3\sin^2 A - \sin^2 A]$ $= \frac{1}{4} [3 \sin A - 3\sin^3 A - \sin^3 A]$ $= \frac{1}{4} [3 \sin A - 4 \sin^3 A]$ $= \frac{1}{4} \sin 3 A = R.H.S.$	1/2 1/2 1/2 1/2 1/2 1/2
	e)	Prove : $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \cos^{-1}\left(\frac{9}{2}\right)$	04
	Ans	$L.H.S. = \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$ $= \tan^{-1} \left[\frac{\frac{1}{7} + \frac{1}{13}}{1 - \left(\frac{1}{7}\right)\left(\frac{1}{13}\right)} \right]$ $= \tan^{-1} \left(\frac{2}{9} \right)$ $R.H.S. = \cot^{-1} \left(\frac{9}{2} \right)$ $\cot^{-1} \left(\frac{9}{2} \right) \neq \cos^{-1} \left(\frac{9}{2} \right)$ $\therefore L.H.S. \neq R.H.S.$ Note: "If Students attempted to solve the question Give appropriate marks."	2 1 1/2 1/2



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5.	a)(ii)	$\therefore \frac{y-5}{8} = \frac{x-2}{-1}$ $\therefore -y+5 = 8x-16$ $\therefore 8x+y-21=0$	1
	b)	Attempt the following:	06
	(i)	Find the acute angle between the lines $3x+2y+4=0$ and $2x-3y-7=0$.	03
	Ans	For $3x+2y+4=0$, $\text{slope } m_1 = \frac{-a}{b} = \frac{-3}{2}$ For $2x-3y-7=0$, $\text{slope } m_2 = \frac{-a}{b} = \frac{-2}{-3} = \frac{2}{3}$ $\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $ $= \left \frac{\frac{-3}{2} - \frac{2}{3}}{1 + \left(\frac{-3}{2}\right)\left(\frac{2}{3}\right)} \right $ $\therefore \tan \theta = \infty$ $\therefore \theta = \tan^{-1}(\infty)$ $\therefore \theta = 90^\circ \quad \text{or} \quad \frac{\pi}{2}$	$\frac{1}{2}$ $\frac{1}{2}$
		OR	
		Consider $m_1 m_2 = \left(\frac{-3}{2}\right)\left(\frac{2}{3}\right)$	1
		$= -1$	1
		$\therefore m_1 m_2 = -1$	
		$\therefore$ Lines are perpendicular	
		$\therefore \theta = 90^\circ \quad \text{or} \quad \frac{\pi}{2}$	1



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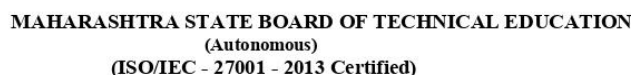
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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)(ii)	Find the distance between lines $3x + 2y = 5$ and $6x + 4y = 6$	03
	Ans	$L_1 : 3x + 2y - 5 = 0$ and $L_2 : 6x + 4y - 6 = 0$ $\therefore L_1 : 6x + 4y - 10 = 0$ and $L_2 : 6x + 4y - 6 = 0$ $\therefore a = 6, b = 4, c_1 = -10$ and $c_2 = -6$	
		$d = \left \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right $	2
		$= \left \frac{-6 + 10}{\sqrt{6^2 + 4^2}} \right $	1
		$= \left \frac{4}{\sqrt{52}} \right $	
		$= 0.555 \quad \text{or} \quad \frac{2}{\sqrt{13}}$	

	c)	Attempt the following:	06
	(i)	A square grassy plot is of side 100 metre. It has a gravel path 10 metres wide all round it on the inside. Find the area of path.	03
	Ans	Area of path = Area of grassy plot – Area of inner square of grassy plot $= (100)^2 - (80)^2$ $= 3600$	2 1
6.	c)(ii)	The volume of cube is 1000 cm^3 . Find its total surface area.	03
	Ans	Let side of cube = l	
		$\therefore$ volume of cube = $l^3 = 1000$	
		$\therefore l = 10$	1
		Total surface area of cube = $6l^2$	
		$= 6(10)^2$	1
		$= 600$	1

		Attempt any two of the following:	12
	a)	Find mean, standard deviation and coefficient of variance of the following data:	



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Q. No.	Sub Q. N.	Answers	Marking Scheme																																										
6.	a)	<u>OR</u> <table><tr><th>Class Interval</th><th>x_i</th><th>f_i</th><th>$f_i x_i$</th><th>x_i^2</th><th>$f_i x_i^2$</th></tr><tr><td>0-10</td><td>5</td><td>3</td><td>15</td><td>25</td><td>75</td></tr><tr><td>10-20</td><td>15</td><td>5</td><td>75</td><td>225</td><td>1125</td></tr><tr><td>20-30</td><td>25</td><td>8</td><td>200</td><td>625</td><td>5000</td></tr><tr><td>30-40</td><td>35</td><td>3</td><td>105</td><td>1225</td><td>3675</td></tr><tr><td>40-50</td><td>45</td><td>1</td><td>45</td><td>2025</td><td>2025</td></tr><tr><td></td><td></td><td>20</td><td>440</td><td></td><td>11900</td></tr></table> Mean $\bar{x} = \frac{\sum f_i x_i}{N}$ $\therefore \bar{x} = \frac{440}{20}$ $\therefore \bar{x} = 22$ S.D. $\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$ $= \sqrt{\frac{11900}{20} - (22)^2}$ $\sigma = 10.54$ Coefficient of variance $= \frac{\sigma}{\bar{x}} \times 100$ $= \frac{10.54}{22} \times 100$ $= 47.91$ -----	Class Interval	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$	0-10	5	3	15	25	75	10-20	15	5	75	225	1125	20-30	25	8	200	625	5000	30-40	35	3	105	1225	3675	40-50	45	1	45	2025	2025			20	440		11900	3 1 1 1
Class Interval	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$																																								
0-10	5	3	15	25	75																																								
10-20	15	5	75	225	1125																																								
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40-50	45	1	45	2025	2025																																								
		20	440		11900																																								



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Q. No.	Sub Q. N.	Answers	Marking Scheme																																
6.	b)	Attempt the following:	06																																
	(i)	Find mean for the following data:																																	
		<table><tr><td>Class-Interval</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr><tr><td>Frequency</td><td>4</td><td>6</td><td>10</td><td>18</td><td>9</td><td>3</td></tr></table>	Class-Interval	10-20	20-30	30-40	40-50	50-60	60-70	Frequency	4	6	10	18	9	3	03																		
	Class-Interval	10-20	20-30	30-40	40-50	50-60	60-70																												
	Frequency	4	6	10	18	9	3																												
	Ans	<table><tr><td>Class</td><td>x_i</td><td>f_i</td><td>$f_i x_i$</td></tr><tr><td>10-20</td><td>15</td><td>4</td><td>60</td></tr><tr><td>20-30</td><td>25</td><td>6</td><td>150</td></tr><tr><td>30-40</td><td>35</td><td>10</td><td>350</td></tr><tr><td>40-50</td><td>45</td><td>18</td><td>810</td></tr><tr><td>50-60</td><td>55</td><td>9</td><td>495</td></tr><tr><td>60-70</td><td>65</td><td>3</td><td>195</td></tr><tr><td></td><td></td><td>50</td><td>2060</td></tr></table>	Class	x_i	f_i	$f_i x_i$	10-20	15	4	60	20-30	25	6	150	30-40	35	10	350	40-50	45	18	810	50-60	55	9	495	60-70	65	3	195			50	2060	2
	Class	x_i	f_i	$f_i x_i$																															
	10-20	15	4	60																															
	20-30	25	6	150																															
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50-60	55	9	495																																
60-70	65	3	195																																
		50	2060																																
	$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N}$ $\therefore \bar{x} = \frac{2060}{50}$ $\therefore \bar{x} = 41.2$	$\frac{1}{2}$																																	
			$\frac{1}{2}$																																
	b)(ii)	The two sets of observation are given below:	03																																
		<table><tr><td>Set-I</td><td>Set-II</td></tr><tr><td>$\bar{x} = 82.5$</td><td>$\bar{x} = 48.75$</td></tr><tr><td>$\sigma = 7.3$</td><td>$\sigma = 8.35$</td></tr></table>	Set-I	Set-II	$\bar{x} = 82.5$	$\bar{x} = 48.75$	$\sigma = 7.3$	$\sigma = 8.35$																											
Set-I	Set-II																																		
$\bar{x} = 82.5$	$\bar{x} = 48.75$																																		
$\sigma = 7.3$	$\sigma = 8.35$																																		
		Which of the two sets is more consistent?																																	



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6.	b)(ii)	Ans Coefficient of variance $V = \frac{\sigma}{x} \times 100$ For set-I $V_1 = \frac{7.3}{82.5} \times 100$ $\therefore V_1 = 8.848$ For set-II $V_2 = \frac{8.35}{48.75} \times 100$ $\therefore V_2 = 17.128$ $\therefore V_1 < V_2$ $\therefore \text{Set-I is more consistent.}$	1 1 1
	c)	Solve the following equations by matrix inversion method : $x + 3y + 2z = 6, 3x - 2y + 5z = 5, 2x - 3y + 6z = 7.$	06
	Ans	Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $ A = \begin{vmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{vmatrix}$ $ A = 1(-12 + 15) - 3(18 - 10) + 2(-9 + 4)$ $ A = -31$ $\therefore A \neq 0$ $\therefore A^{-1} \text{ exists}$	1



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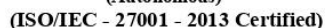
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6.	c)	$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} -2 & 5 \\ -3 & 6 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ -3 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} 3 & 8 & -5 \\ 24 & 2 & -9 \\ 19 & -1 & -11 \end{bmatrix}$ $\text{Matrix of cofactors} = \begin{bmatrix} 3 & -8 & -5 \\ -24 & 2 & 9 \\ 19 & 1 & -11 \end{bmatrix}$ OR $C_{11} = + \begin{vmatrix} -2 & 5 \\ -3 & 6 \end{vmatrix} = 3, \quad C_{12} = - \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} = -8, \quad C_{13} = + \begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} = -5$ $C_{21} = - \begin{vmatrix} 3 & 2 \\ -3 & 6 \end{vmatrix} = -24, \quad C_{22} = + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 2, \quad C_{23} = - \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = 9$ $C_{31} = + \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} = 19, \quad C_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 1, \quad C_{33} = + \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = -11$ $\text{Matrix of cofactors} = \begin{bmatrix} 3 & -8 & -5 \\ -24 & 2 & 9 \\ 19 & 1 & -11 \end{bmatrix}$ $\text{Adj. } A = \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{Adj. } A$ $= \frac{1}{-31} \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$	1 1 2 ½ ½



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