

ENGINEERING MATHEMATICS

Topic1:COMPLEX NUMBER

1) $Z_1 = 4+3i$ and $Z_2 = 2-3i$ Then find i) $Z_1 + Z_2$ ii) $Z_1 - Z_2$ iii) $Z_1 \cdot Z_2$ iv) $\frac{Z_1}{Z_2} \cdot v) \frac{1}{z_1} + \frac{1}{z_2}$

2) Express the following complex number in $x+iy$ form-

a) $\frac{2-\sqrt{3}i}{1+i}$ b) $\frac{2+i}{2-i}$ c) $\frac{2+i}{(2+i)(3+i)}$ d) $\frac{1}{3+5i} + \frac{1}{4-2i}$.

3) If $Z = \frac{1-i}{1+i}$ then show that z is purely imaginary.

4) If $Z_1 = 1+2i$ and $Z_2 = 2-3i$ then express $\frac{Z_2}{Z_1}$ in $a + ib$ form.

5) Find modulus and amplitude of i) $\frac{Z_1}{Z_2}$ IF $Z_1 = 2+2\sqrt{3}i$ and $Z_2 = -1 + \sqrt{3}i$.

ii) $Z = 1 - \sqrt{3}i$ iii) $\frac{(1+i)^2}{1-i}$

6) Express the following complex number in polar form

i) $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$ ii) $1 - \cos \alpha + i \sin \alpha$ iii) $-\sqrt{3} + i$ IV) $-2 - 2\sqrt{3}i$ v) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

VI) $2\sqrt{2}(1 + i)$ vii) $3\sqrt{2}(-1 + i)$

7) Using De-moivre's theorem simplify

i) $\frac{(\cos 5\theta - i \sin 5\theta)^2 (\cos 7\theta + i \sin 7\theta)^{-3}}{(\cos 4\theta - i \sin 4\theta)^9 (\cos \theta + i \sin \theta)^5}$ ii) $\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 5\theta - i \sin 5\theta)^{\frac{4}{5}}}{(\cos \frac{9}{2}\theta + i \sin \frac{9}{2}\theta)^{\frac{2}{3}} (\cos \frac{4}{5}\theta + i \sin \frac{4}{5}\theta)^{10}}$

8) Show that $(1 + i)^8 + (1 - i)^8 = 32$.

9) Using Euler's formula prove that:

i) $\sin^2 \theta + \cos^2 \theta = 1$ ii) $\sin 2\theta = 2\sin \theta \cdot \cos \theta$ iii) $\cosh^2 \theta - \sinh^2 \theta = 1$. iv) $1 + \tan^2 \theta = \sec^2 \theta$

10) If $\sin(A + iB) = x + iy$ then prove that

i) $\frac{X^2}{\cosh^2 B} + \frac{Y^2}{\sinh^2 B} = 1$ ii) $\frac{X^2}{\sin^2 A} - \frac{Y^2}{\cos^2 A} = 1$

11) If $\cos(A + iB) = x + iy$ then prove that

i) $\frac{X^2}{\cosh^2 B} + \frac{Y^2}{\sinh^2 B} = 1$ ii) $\frac{X^2}{\cos^2 A} - \frac{Y^2}{\sin^2 A} = 1$.

12) Separate into real and imaginary parts i) $\sin(x + iy)$ ii) $\cosh(x + iy)$ iii) $\sinh(x - iy)$

13) Separate into real and imaginary parts $\frac{(1+i^3)^2}{(3-i)^3}$.

14) Find cube root of unity.

15) If $(3 + i)x + (1 - i)y = 1 + 7i$, find x and y .

16) Find real and imaginary part of $z + z^{-1}$.

17) Using Euler's formula prove that $\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$.

18) If $3 + 4bi = 5a + 6i$ find a and b .

19) Using De-Moivre's thm. prove that $(1 + i)^8 + (1 - i)^8 = 32$.

20) Using De-Moivre's theorem simplify $\frac{(\cos \theta - i \sin \theta)^6 (\cos 5\theta - i \sin 5\theta)^{-2}}{(\cos 8\theta + i \sin 8\theta)^{\frac{1}{2}}}$.

TOPIC2:Function

- 1) Define even and odd function
- 2) If $f(x) = x^3 - 5x^2 - 4x + 20$ then show that $f(0) = -2f(3)$
- 3) If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$
- 4) If $f(x) = x^2 + 3$ then find the value of x for which $f(x) = f(2x+1)$
- 5) Show that $3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x$ is even function.
- 6) If $f(x) = \sin x$ $g(x) = \cos x$ show that $f(x+y) = f(x)g(y) + g(x)f(y)$
- 7) If $f(x) = \cos x$ show that $f(3x) = 4f^3(x) - 3f(x)$
- 8) If $f(x) = \tan x$ show that $f(2x) = \frac{2f(x)}{1 - (f(x))^2}$
- 9) If $f(x) = 50\sin(100\pi t + 0.4)$ show that $f\left(\frac{1}{50} + t\right) = f(x)$
- 10) If $f(x) = \log\left(\frac{x}{x-1}\right)$ then prove that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$
- 11) If $f(x) = \frac{x+5}{3x-4}$ and $t = \frac{5+4x}{3x-1}$ then show that $f(t) = x$
- 12) If $y = f(x) = \frac{x+1}{x-1}$ then show that $x = f(y)$
- 13) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$
- 14) If $f(x) = \frac{2x+5}{3x-4}$ and $g(x) = \frac{5+4x}{3x-2}$ then show that $f \circ g(x) = x$
- 15) If $y = f(x) = \frac{2x-3}{3x-2}$ then show that $x = f(y)$
- 16) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ then show that $f(t) = x$

- 17) if $f(x) = \log x$, $\phi(x) = x^3$ show that $f[\phi(x)] = 3f(x)$.
- 18) if $f(x) = 3x^4 + x^2 + 5 - 3 \cos x + 3 \sin^2 x$, show that $f(x) + f(-x) = 2f(x)$.
- 19) if $f(x) = x^2 + 4$, solve $f(x+1) - f(x-1) - 12 = 0$.
- 20) if $f(x) = x - \frac{1}{x}$ then prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.
- 21) if $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then show that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$.
- 22) if $f(x) = 16^x + \log_4 x$ then find $f\left(\frac{1}{2}\right)$ and $f\left(\frac{1}{4}\right)$.
- 23) if $f(x) = \frac{x^2+9}{\sqrt{x-3}}$ find $f(4) + f(5)$.
- 24) if $f(x) = \log(x + \sqrt{1+x^2})$, show that the function is odd.

TOPIC3: NUMERICAL METHOD'S

- 1) Using Bisection method find the approximate root of following equation's-
- $x^2 - 2x - 5 = 0$ upto three iterations.
 - $x^2 - \log_e x = 1.8$ upto three iterations.
 - $x^3 - 5x - 3 = 0$ root lies between 2 & 3 upto three iterations.
 - $x^3 - 9x - 5 = 0$ in the interval(3,4).
 - $\sqrt{40}$. carry out three iterations.
 - Find first two roots of equation $x^3 - 2x - 5 = 0$.
- 2) Using Regula Falsi / False position method find the approximate root of following equations-
- $x^2 - 2x - 9 = 0$ up to three iterations.
 - $x^3 + 2x - 50 = 0$
 - $xe^x - 8 = 0$
 - $x^3 + 3x - 20 = 0$ between 2 & 3 upto three iterations.
 - $x - \log x = 6$
 - $x^3 - 9x + 1 = 0$.
- 3) By using Newton Raphson method find the root of following equations upto three iteration—
- $x^4 - x - 5 = 0$
 - $x^3 - x - 1 = 0$ by taking initial root 1.
 - $\sqrt[3]{100}$

d) $x^3 - 20 = 0$

e) $\sqrt{10}$

f) $x^3 - 4x + 1 = 0$ in $(1,2)$.

4) Using Gauss elimination method solve the following equations—

a) $2x + 5y - 3z = 22, x - 2y + 4z = 3, 6x - 7y + z = 10.$

b) $x + y - z = 3, x - y + 4z = 4, 2x + y + z = 7.$

c) $3x + 4y + z = -3, x + 2y + z = -1, 2x - y + z = 6.$

d) $3x - 4y - 6z = 4, 3x + 12y + 9z = -2, 9x + 8y + 3z = 3.$

5) Solve the following equations using Jacobi's method

a) $10x + y + 2z = 9, -2x + 10y + 3z = 5, 3x - y + 10z = -8.$

b) $20x - 3y + z = 17.5, 4x + 15y - 5z = 16.5, x - 7y + 10z = 10.5.$

c) $9x - 5y + z = 30, 3x + 10y + 2z + 7 = 0, 6x + 5y + 12z - 18 = 0.$

d) $30x + 10y - 3z = 96, 4x + 25y + 2z = 53, 2x + 5y + 20z = -6.$

e) $12x + 4y + 3z = 19, 5x + 12y + 2z = 19, 7x + y + 12z = 20.$

6) Solve using Gauss Seidal method following equations---

a) $10x + 3y + 2z = 2, 5x + 10y + z = 18, 7x + 2y + 10z = 27.$

b) $-15x + 9y + 4z = 2, 6x - 14y + 3z = 5, 2x + 5y - 13z = 6.$

c) $x + 7y - 3z = -22, 5x - 2y + 3z = 18, 2x - y + 6z = 22$

d) $5x - y = 9, x - 5y + z = -4, y - 5z = 6$ using initial approximations $x_0 = 1.5,$

$y_0 = 0.5, z_0 = -0.5.$

e) $8x + y + z = 10, 3x - 8y + 5z = 0, 7x + 8z = 15,$ taking initial approximations as

$x_0 = y_0 = z_0 = 0.75.$

TOPIC4: LIMIT'S

a) Evaluate: $\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$

b) Evaluate: $\lim_{x \rightarrow 1} \frac{x-1}{2x^2-7x+5}$

c) Evaluate : $\lim_{x \rightarrow -5} \frac{2x^2+9x-5}{x+5}$

d) Evaluate: $\lim_{x \rightarrow 5} \frac{x^3-125}{x^2-7x+10}$

e) Evaluate: $\lim_{x \rightarrow 5} \frac{x^3-8}{x^2-4}$

f) Evaluate: $\lim_{x \rightarrow 2} \frac{x^3-6x^2+11x-6}{x^2-6x+8}$

g) Evaluate: $\lim_{x \rightarrow 1} \frac{x^4-3x^2+2}{x^3-5x^2+3x+1}$

h) Evaluate: $\lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{4}{x^2-2x} \right)$

i) Evaluate: $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{2}{x^2-4x+3} \right)$

j) Evaluate: $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9x}{x^3-27} \right)$

k) Evaluate: $\lim_{x \rightarrow 2} \left(\frac{1}{x-3} - \frac{3}{x^2-3x} \right)$

l) Evaluate: $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^3-2x^2} \right)$

m) Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

n) Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

o) Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x}$

p) Evaluate: $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2}$

q) Evaluate: $\lim_{x \rightarrow 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$

r) Evaluate: $\lim_{x \rightarrow 5} \frac{x-5}{\sqrt{6x-5}-\sqrt{4x+5}}$

s) Evaluate: $\lim_{x \rightarrow 7} \frac{4-\sqrt{9+x}}{1-\sqrt{8-x}}$

t) Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{\sqrt{x+9}-3}$

a) Evaluate: $\lim_{x \rightarrow a} \frac{x^9 - a^9}{x - a}$

Limit's form: $\left[\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} \right] = na^{n-1}$

b) Evaluate: $\lim_{x \rightarrow a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x^{\frac{7}{2}} - a^{\frac{7}{2}}}$

c) Evaluate: $\lim_{x \rightarrow 3} \frac{x^5 - 243}{x^3 - 27}$

d) Evaluate: $\lim_{x \rightarrow a} \frac{x^3 - a^3}{x^2 - a^2}$

e) Evaluate: $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$

f) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^5 - 32}$

a) Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$

put $x = \frac{1}{t} \rightarrow t = \frac{1}{x}$ and $x \rightarrow \infty, t \rightarrow 0$.

b) Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1})$

c) Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x - 1})$

d) Evaluate: $\lim_{x \rightarrow \infty} x(\sqrt{x^2 + 4} - \sqrt{x^2 + 1})$

e) Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 7x} - x)$

f) Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 16} - \sqrt{x^2 + 16})$

g) Evaluate: $\lim_{x \rightarrow \infty} \sqrt{x + 2}(\sqrt{x^2 + 4} - \sqrt{x})$

h) Evaluate: $\lim_{x \rightarrow \infty} \frac{1+2+3+\dots+n}{n^2}$

a) Evaluate: $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

as $(\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1), (\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1) (\lim_{x \rightarrow 0} \frac{\sin kx}{kx} = 1)$

b) Evaluate: $\lim_{x \rightarrow 0} \frac{\tan 7x}{\sin 5x}$

c) i) Evaluate: $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{x^2}$

ii) Evaluate: $\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{x^2}$

d) Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3 \tan^2 x}$

e) Evaluate: $\lim_{x \rightarrow 0} \frac{\cos x}{x - \frac{\pi}{2}}$

put $x = \frac{\pi}{2} + h, \therefore x - \frac{\pi}{2} = h, \text{ as } x \rightarrow \frac{\pi}{2}, h \rightarrow 0$.

f) Evaluate: $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

g) Evaluate: $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$

Formulae: $\left(\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a, \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1, \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e, \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e\right)$

a) Evaluate: $\lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$

b) Evaluate: $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^x$

c) Evaluate: $\lim_{x \rightarrow 0} \left(1 + \frac{3x}{2}\right)^{\frac{1}{x}}$

d) a) Evaluate: $i) \lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$ ii) $\lim_{x \rightarrow 0} \frac{3^x - 5^x}{x}$

e) Evaluate: $\lim_{x \rightarrow 0} \frac{7^x - 1}{\sin x}$

f) Evaluate: $\lim_{x \rightarrow 0} \frac{5^x + 3^x - 2^x - 1}{x}$

g) Evaluate: $\lim_{x \rightarrow 0} \frac{8^x - 4^x - 2^x + 1}{x^2}$

h) Evaluate: $\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

i) Evaluate: $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \sin x}$

j) Evaluate: $i) \lim_{x \rightarrow 0} \frac{5^x + 3^x - 2^x - 1}{x}$ $ii) \lim_{x \rightarrow 0} \frac{4^x + 4^{-x} - 2}{x \sin x}$

k) Evaluate: $\lim_{x \rightarrow 0} \frac{5^{\sin x} - 1}{\tan x}$

l) Evaluate: $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{3x}\right)^x$

m) Evaluate: $\lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-2}\right)^{2x}$

n) Evaluate: $\lim_{x \rightarrow 0} \frac{\log(7+x) - \log(7-x)}{x}$ as $\left(\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}\right) = 1, \left(\lim_{x \rightarrow 0} \frac{\log(1+ax)}{x}\right) = a$

o) Evaluate: $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x}$

p) Evaluate: $\lim_{x \rightarrow 0} \frac{\log(5+x) - \log(5-x)}{x}$

q) Evaluate: $\lim_{x \rightarrow 5} \frac{\log x - \log 5}{x - 5}$

r) Evaluate: $i) \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x - 3}$ $ii) \lim_{x \rightarrow 2} \frac{\log x - \log 2}{x - 2}$

s) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

TOPICS: DERIVATIVE'S

a) Using first principle of derivative find derivative of following functions:

1. e^x 2. $3x^2+4$ 3. \sqrt{x} 4. $\log x$ 5. $\sin x$ 6. $\cos x$ 7. $\tan x$ 8. K .

b) If u and v are differentiable functions of x and $y = u + v$, then prove that: $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$.

c) If u and v are differentiable functions of x and $y = u - v$, then prove that: $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$.

d) If u and v are differentiable functions of x and $y = u \cdot v$, then prove that : $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$.

e) If u and v are differentiable functions of x and $y = \frac{u}{v}$, then prove that : $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$.

f) Find $\frac{dy}{dx}$, if i) $y = x^{10} + 10^x + e^x$ ii) $y = (x + 1)(x + 2)$. iii) $y = \sec x \cdot \tan x$ iv) $y = e^x \cdot \tan x$

v) $y = \frac{x+1}{x-1}$ vi) $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$ vii) $y = \frac{\sin x}{1-\cos x}$. viii) $y = \frac{\sin x + \cos x}{\cos x - \sin x}$ ix) $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

f) Differentiate w. r . t . x

i) $\sin(x^2 + 5)$. ii) $\sec(4x - 3)$ iii) $e^{3x} \cos 2x$ iv) $\log(\sin x)$ v) $\tan(x \cdot e^x)$ vi) $(x^2 + x + 1)^4$

g) Find $\frac{dy}{dx}$ If i) $y = \log(3x^2 + 2x + 1)$ ii) $y = e^{\tan x}$ iii) $y = \sin^2[\log(2x + 3)]$ iv) $y = \log(\log x)$.

v) $y = \cos(x^2 e^x)$ vi) $y = \cos^{-1}(7x)$ vii) $y = e^{3x} + \tan^4 x$ viii) $y = e^{\sin x} \cos 4x$.

h) Find $\frac{dy}{dx}$ if $x^3 + y^3 + 4x^3y = 0$

i) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 4xy$

j) Find $\frac{dy}{dx}$ if $x^2 + 3y + y^2 = 5$ at $(1,1)$.

k) Find $\frac{dy}{dx}$ if $x^2 + y^2 = 4xy$.

l) Find $\frac{dy}{dx}$, if $\sin y = \log(x + y)$

m) Find $\frac{dy}{dx}$ if $x^2 + y^2 + xy - y = 0$ at (1,2) .

n) if $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$ at the point $(\frac{3a}{2}, \frac{3a}{2})$.

o) if $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

p) if $e^y = e^x$, then find $\frac{dy}{dx}$

q) Find $\frac{dy}{dx}$ if $x^2 + y^2 = 25$.

r) Find $\frac{dy}{dx}$ if $x^2 + 3xy - y^2 = 11$ at (1,2) .

a) if $y = e^{2x} \cdot \sec x \cdot \tan 2x$, Find $\frac{dy}{dx}$.

b) i) if $y = \frac{(x+1)^4}{(x-1)^3(x+4)^7}$, Find $\frac{dy}{dx}$. ii) if $y = \frac{(x-2)^3(x-3)^5}{(x+1)^4}$, Find $\frac{dy}{dx}$.

c) Find $\frac{dy}{dx}$, if $y = x^x$.

d) Differentiate $(\sin x)^{\tan x}$ w.r. to x .

e) Differentiate $(\tan x)^{\cot x}$ w.r. to x .

f) Differentiate w.r. to x . i) $(\log x)^{\log x}$ ii) $(\sin x)^{\cos x}$.

a) Find $\frac{dy}{dx}$, if $x = 3at^2$ and $y = 2at^3$.

b) Find $\frac{dy}{dx}$, if $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$.

c) Find $\frac{dy}{dx}$, if $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$.

d) Find $\frac{dy}{dx}$, if $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$.

e) Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$.

f) Find $\frac{dy}{dx}$, if $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$.

a) Differentiate $\log(x \cdot \sin x)$ w.r. to $\frac{1}{x}$.

b) Differentiate $\tan^{-1}(4x)$ w.r. to e^{6x} .

c) Differentiate $(\log x)^x$ w.r. to 5^{4x} .

d) Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r. to $\sin^{-1}(2x\sqrt{1-x^2})$.

e) Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r. to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

f) Differentiate $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r. to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

a) if $xy = a$, show that $x \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$.

b) if $y = \sin 5x - 3 \cos 5x$, show that $\frac{d^2y}{dx^2} + 25y = 0$.

c) if $y = \sin^{-1} x$, show that $(1-x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$.

d) if $y = 2 \cos(\log x) + 3 \sin(\log x)$, show that $x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + y = 0$.

e) if $y = e^{m \cdot \sin^{-1} x}$, show that $(1-x^2) \cdot \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$.

f) if $y = ae^{2x} + be^{-2x}$, show that $\frac{d^2y}{dx^2} - 4y = 0$.

g) if $y = A \cos x + B \sin x$, show that $\frac{d^2y}{dx^2} + y = 0$.

h) if $y = \cos(\log x)$, show that $x^2 \cdot \frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} - y = 0$.

i) if $y = 3 \sin 4x - 5 \cos 4x$, show that $\frac{d^2y}{dx^2} + 16y = 0$.

j) if $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + y = 0$.
